Homework #2 (Due: 11/3)

1. Problem 4-11. (Gelb)
2. Consider the second-order system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\zeta\omega \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) \qquad \text{(H2.1)}$$

where $\omega = 6\, rad/\sec$ is the natural frequency of the system, and $\zeta = 0.16$ is the damping ratio. The input $w(t)$ is continuous-time white Gaussian noise with zero mean and a variance of 0.01. Measurements of the first state are taken every 0.5 sec:

$$z(n) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}(n) + v(n) \qquad \text{(H2.2)}$$

where $v(n)$ is discrete-time white Gaussian noise with zero mean and a variance of 10^{-4}. The initial state, estimate, and covariance are:

$$\underline{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad \hat{\underline{x}}(0) = \underline{x}(0); \quad P(0) = \begin{bmatrix} 10^{-5} & 0 \\ 0 & 10^{-2} \end{bmatrix}. \qquad \text{(H2.3)}$$

(1) Discretize the system equation.

(2) Using MATLAB, generate state variable samples and noisy observations for 10 sec.

(3) Implement the discrete Kalman filter to estimate the state of this system. Plot the true states $x_i(n)$ and estimated states $\hat{x}_i(n)$ along with $1-\sigma$ bound. Apply various values of the variance of $w(t)$ in the filter model and compare with the one applied the true variance. What can you conclude?

3. Let

$$\underline{x}_{k+1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \underline{x}_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w_k,$$
$$z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}_k + v_k, \qquad \text{(H2.4)}$$

with $w_k \sim (0,1)$, $v_k \sim (0,1)$ white and uncorrelated.

(1) Solve the algebraic Riccati equation to determine the optimal steady-state Kalman filter. Find the filter poles and the transfer function from data z_k to estimate $\hat{\underline{x}}_k$.

(2) Find the Wiener filter and compare to (1).