Homework #3 (Due: 11/22)

1. Consider the second-order system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\zeta\omega \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

where $\omega = 6\, rad/\sec$ is the natural frequency of the system, and $\zeta = 0.16$ is the damping ratio. The input $w(t)$ is continuous-time white noise with a variance of 0.01. Measurements of the first state are taken every 0.5 sec

$$z(t_k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}(t_k) + v(t_k)$$

where $v(t_k)$ is discrete-time white noise with a variance of 10^{-4}. The initial state, estimate, and covariance are

$$\underline{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{\hat{x}}(0) = \underline{x}(0)$$

$$P(0) = \begin{bmatrix} 10^{-5} & 0 \\ 0 & 10^{-2} \end{bmatrix}.$$

Repeat the steps in Homework #2 to discretize the system equation and to generate state variable samples and noisy observations for 10 sec using MATLAB. Then, implement the fixed-interval smoother for 10 sec (20 time steps). Plot the variance of the estimation error of the first state for the forward filter and the smoother on a single plot. Do the same for the second state. Discuss about the results. (Don’t forget to attach m-codes to your report.)