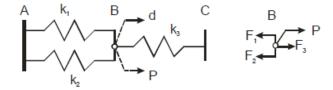
## Problem 4.1 Simple hyperstatic bars- displacement method solution



Three axially loaded bars, each of length L and all constructed from a material of elasticity modulus E, are arranged as shown in fig. 4.9. Two bars are connected in parallel and one of these has a cross-sectional area that is twice that of the other. A third bar is connected in series at the common point. An axial load, P, is applied at the junction of the three bars. Using the displacement method, determine (1) the displacement, d, of the connecting point between the three bars and (2) the forces in each of the three bars.

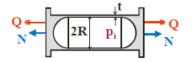
## Problem 4.2. Simple hyperstatic bars – force method solution

Solve problem 4.1 using the force method.

# Problem 7.1. Torsion of a bimetallic bar

A circular bar is constructed by bonding an aluminum shell around a solid steel cylinder. The radius of the steel cylinder is  $R_s = 10$ mm, and the outer radius of the aluminum shell is  $R_A = 20$ mm. The overall length of the bar is given by L = 1m, and a torque T = 1kN-m is applied at the ends. The shear moduli for the aluminum and steel are  $G_A = 28$ GPa and  $G_s = 76$ GPa, respectively. (1) Find the maximum shear stress in the steel and in the aluminum. (2) Determine the total twist angle of the bar. (3) Determine the torsional stiffness. (4) Find the allowable torque for a safety factor of 2 when the yield stresses for both materials is 300MPa.

#### Problem 7.6. Pressure vessel subjected to combined loading



The experimental set-up depicted in fig. 7.13 is aimed at studying the behavior of materials under complex stress states. A thin-walled pressure vessel of radius R=11mm and thickness t = 2.0mm is subjected to an internal pressure pi. At the same time, a normal force, N, and a torque, Q, are applied to the sample. In a specific experiment, the applied normal force is N = 16 kN and the internal pressure pi = 20 MPa. The applied torque is slowly increased. The first permanent deformations are observed at the outer surface of the sample when Q = 120 N-m.

- (1) Find the yield stress for the material if it is assumed to follow von Mises' yield criterion.
- (2) Find the yield stress for the material if it is assumed to follow Tresca's yield criterion.
- (3) Find and plot the yield surface in the space defined by the three loading components, the internal pressure, the applied axial force, and the applied torque.

# Problem 7.7. Beam with circular section under bending and torsion

Consider a cantilevered beam of length L = 1 m with a circular cross-section of inner radius  $R_i = 45$  mm and outer radius  $R_o = 50$  mm. The beam is subjected to a tip torque Q = 7 kN-m and a tip transverse load P. Find the maximum allowable transverse load Pmax if the allowable stress for the material is  $\sigma_{allow} = 250$  MPa.