2013-2, 해양환경정보시스템

Homework 0 Solution

1. (30%) Describe the following terms:

(1) Ideal fluid: Inviscid $\left(\nu = 0\right)$ and Incompressible $\left(\dfrac{d\rho}{dt} = 0\right)$

(2) Newtonian flow: Fluid which satisfies with $\tau_{ij} \propto \dfrac{\partial u_l}{\partial x_m} \Rightarrow \tau_{ij} = \alpha_{ijlm} \dfrac{\partial u_l}{\partial x_m}$ where α_{ijlm} is a coefficient tensor. For incompressible Newtonian fluid, $\tau_{ij} = \mu \left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i} \right)$

(3) Vorticity: $\vec{\omega} = \nabla \times \vec{u}$

(4) Velocity potential: If (i) ideal fluid $\left(\nu = 0,\ \dfrac{d\rho}{dt} = 0\right)$ and (ii) irrotational flow $\left(\nabla \times \vec{u} = 0\right)$, we can define the velocity potential ϕ

$$\vec{u} = \nabla \phi$$

(5) Streamline: A line everywhere tangent to the fluid velocity $\vec{u}$ at a given time.

In an Eulerian description, it would be a 'snapshot' of the flow.

Streakline: Instantaneous locus of all particles that pass a given point. In an Eulerian description, it would be a 'snapshots' of certain particles.

Pathline: The trajectory of a given particle P in time. The photograph analogy would be a long time exposure of a given particle.

Timeline: a set of adjacent fluid particles that were marked at the same (earlier) instant in time

(6) Added mass: The hydrodynamic force due to forced motion is in phase with the acceleration. Thus it acts as an apparent mass adding to the mass of the body. That is 'Added mass'.

2. (20%) Write the velocity potential of the following singularity.

(1) 2-D source: $\phi = \frac{m}{2\pi}\ln r$, **2-D dipole:** $\phi = \frac{m}{2\pi}\frac{\cos\theta}{r}$

(2) 3-D source: $\phi = -\frac{m}{4\pi r}$, **3-D dipole:** $\phi = \frac{m}{4\pi}\frac{\cos\theta}{r^2}$

3. (20%) In a 3-D fluid domain, write the following equations. Use your own definition as needed.

(1) Laplace equation: $\nabla^2\phi = 0$

(2) Bernoulli's equation: $\frac{\partial\phi}{\partial t} + \frac{1}{2}\left|\nabla\phi\right|^2 + \frac{p}{\rho} + gz = \mathrm{f(t)}$

(3) Navier-Stokes equation: $\frac{D\vec{u}}{Dt} = \frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \vec{f}$

4. (30%) Let's consider a 1-D vibration system consisted of a mass m and spring k.

(1) Derive equation of motion: $\vec{F} = m\vec{a} \Leftrightarrow m\ddot{x} = -f_s \Leftrightarrow m\ddot{x} + f_s = 0 \Leftrightarrow m\ddot{x} + kx = 0$

(2) What is the natural frequency of this system?

$$m\ddot{x} + kx = 0 \Rightarrow x = ce^{st} \Rightarrow ms^2 + k = 0 \Rightarrow s = \pm i\sqrt{\frac{k}{m}} = \pm i\omega_N \Rightarrow \omega_N = \sqrt{\frac{k}{m}}$$

5. (20%) Let's consider a 2-D source near a vertical wall, as shown in the right figure. Its coordinate is (x,y)=(-1,0).

(1) Show that the wall boundary condition satisfies if one additional point source locates at (1,0), (1,-1).

$$\phi_1 = \frac{m}{2\pi}\ln r \ \ where\ r = \sqrt{(x+1)^2 + y^2}$$

Source at (1,0): $$\phi = \frac{m}{2\pi}\ln\sqrt{(x+1)^2 + y^2} + \frac{m}{2\pi}\ln\sqrt{(x-1)^2 + y^2}$$

$$u = \frac{\partial \phi}{\partial x} = \frac{m}{2\pi}\left\{\frac{x+1}{(x+1)^2 + y^2} + \frac{x-1}{(x-1)^2 + y^2}\right\}$$

At x=0: $$u\big|_{x=0} = \frac{\partial \phi}{\partial x}\bigg|_{x=0} = \frac{m}{2\pi}\left\{\frac{1}{1+y^2} + \frac{-1}{1+y^2}\right\} = 0$$

Source at (1,-1): $$\phi = \frac{m}{2\pi}\ln\sqrt{(x+1)^2 + y^2} + \frac{m}{2\pi}\ln\sqrt{(x-1)^2 + (y+1)^2}$$

$$u = \frac{\partial \phi}{\partial x} = \frac{m}{2\pi}\left\{\frac{x+1}{(x+1)^2 + y^2} + \frac{x-1}{(x-1)^2 + (y+1)^2}\right\}$$

At x=0: $$u\big|_{x=0} = \frac{\partial \phi}{\partial x}\bigg|_{x=0} = \frac{m}{2\pi}\left\{\frac{1}{1+y^2} + \frac{-1}{1+(y+1)^2}\right\} \neq 0$$

(2) Obtain velocity potential, velocity, and pressure at two locations: i) x=0, y=1, ii) x=-0.5, y=0.5.

$$\phi = \frac{m}{2\pi}\ln\sqrt{(x+1)^2 + y^2} + \frac{m}{2\pi}\ln\sqrt{(x-1)^2 + y^2}$$

$$u = \frac{\partial\phi}{\partial x} = \frac{m}{2\pi}\left\{\frac{x+1}{(x+1)^2 + y^2} + \frac{x-1}{(x-1)^2 + y^2}\right\}$$

$$v = \frac{\partial\phi}{\partial y} = \frac{m}{2\pi}\left\{\frac{y}{(x+1)^2 + y^2} + \frac{y}{(x-1)^2 + y^2}\right\}$$

$$p = -\frac{1}{2}\rho\left|\nabla\phi\right|^2 = -\frac{1}{2}\rho\left\{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right\}$$

i) x = 0, y = 1

$$\phi = \frac{m}{2\pi}\ln\sqrt{2} + \frac{m}{2\pi}\ln\sqrt{2} = \frac{m}{2\pi}\ln\sqrt{4} = \frac{m}{\pi}\ln\sqrt{2}$$

$$u = 0$$

$$v = \frac{m}{2\pi}$$

$$p = -\frac{1}{2}\rho\left(\frac{m}{2\pi}\right)^2$$

ii) x = -0.5, y = 0.5

$$\phi = \frac{m}{2\pi}\ln\sqrt{5/4}$$

$$u = \frac{\partial\phi}{\partial x} = \frac{m}{5\pi}$$

$$v = \frac{3m}{5\pi}$$

$$p = -\frac{1}{2}\rho\left(\frac{m}{2\pi}\right)^2\frac{8}{5} = -\frac{1}{2}\rho\frac{2m^2}{5\pi^2}$$