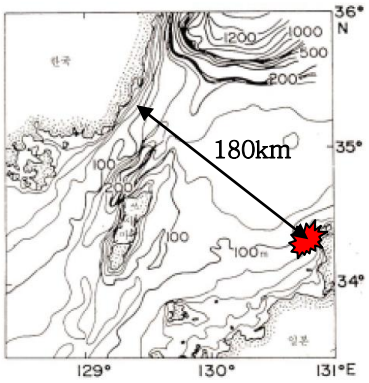
Homework #2

Course: 414.311A

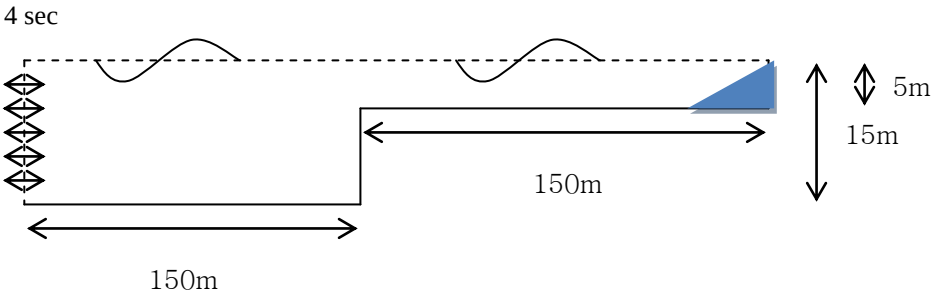
Due on
November 18, 2013

1. (10%) Using the dispersion relation, show that $d\omega/dk$ is the group velocity.

2. (10%) The left map shows the depth contour of Korea Strait. Let's assume that an earthquake occurred in the coastal area of Japan across the Strait, and there is a possibility of tsunami occurrence. If the shortest distance in the Korea Strait is about 180km and the averaged depth is 100m, how long does it take for the tsunami to hit the Korean costal area? Ignore the bottom friction, depth variation, and current effects.



3. (20%) There is a water tank of total length 300m, as shown in the below figure. A wave-maker started to generate waves with the period of 4 sec.

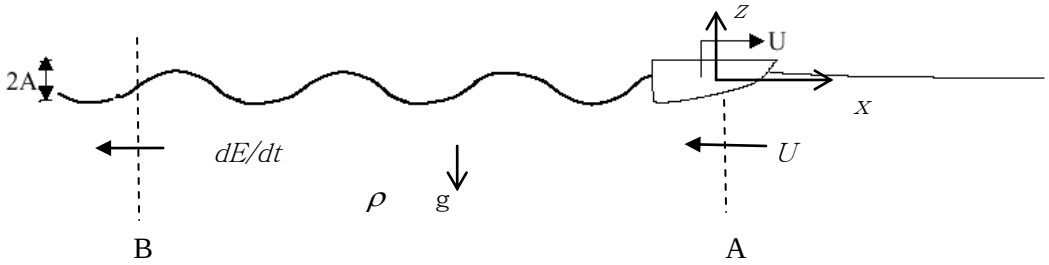


(1) Estimate the time when the first wave arrive the step at 150m far from the wave-maker and the end of the tank.

(2) After the reflection of the first wave in the underwater step at 150m, 30% of the wave energy is reflected to left and 70% transmitted to right. Write the wave elevation (as an equation) of the generated and reflected waves before it reflects again on the wave maker. What are the maximum and minimum amplitudes?

(3) How long does it take for the first transmitted wave reaches to the end of water tank and what is its amplitude?

4. (30%) Consider a ship moving with a constant speed U in 2D fluid domain. Regular waves are generated behind the ship. Assume potential flows around the body.



(1) Write the wave number and wave frequency as a function of speed U and gravitational constant g.

(2) What is the energy flux following by the wave on section B?

(3) Ship is experiencing wave resistance D. What is energy flux into the water by the ship (on section A in above figure)? Considering energy conservation, write the wave resistance D as a function of wave amplitude speed, fluid density, and gravitational constant.

(4) Let's assume that the ship wave can be considered as a sum of the waves generated by a source at ship bow and a sink at ship stern with same strength. If a moving point singularity generates regular waves with amplitude written to $a\cos(kx)$ where *k* is wave number, find the distance of two singularities (i.e. ship length) which provides minimum wave height behind the ship.

5. (30%) Consider the ISSC spectrum, s.t.

$$S_\eta(\omega) = \frac{0.11}{2\pi} H_{1/3}{}^2 T_1 \left(\frac{\omega T_1}{2\pi}\right)^{-5} e^{-0.44\left(\frac{\omega T_1}{2\pi}\right)^{-4}}$$

We will consider $H_{1/3}$=5m, and T_1=9sec.

(1) Discretize this spectrum into 10 components from 0.2 rad/sec to 2.0 rad/sec. Obtain a set of frequencies and the wave amplitudes of 10 components.

(2) The phase of each component can be randomly picked from 0 to 2π. By using the wave amplitude and frequency which are obtained in (1), and choosing randomly the phase of each component, create a wave signal of 100-sec time window with time segment of 0.1sec. That is, make a graph of wave elevation from 0 to 100 seconds. (MS Excel program will be useful for this task. You can use any other program e.g. MATLAB.)