SOLUTION

PROBLEM 12.69

$\dot{\theta} = 10$ rad/s, $\ddot{\theta} = 0$, $b = 225$ mm $\quad m = 0.11$ kg

Kinematics: $r = \dfrac{b}{\cos\theta}$, $\quad \dot{r} = \dfrac{b\sin\theta}{\cos^2\theta}\dot{\theta}$

$$\ddot{r} = \frac{b\sin\theta}{\cos^2\theta}\ddot{\theta} + \frac{b\left[\left(\cos^2\theta\right)\left(\cos\theta\right) - \left(\sin\theta\right)\left(2\cos\theta\right)\left(-\sin\theta\right)\right]}{\cos^4\theta}\dot{\theta}^2$$

$$= \frac{b\left(1+\sin^2\theta\right)}{\cos^3\theta} \quad \text{with} \quad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{b\left(1+\sin^2\theta\right)}{\cos^3\theta}\dot{\theta}^2 - \frac{b}{\cos\theta}\dot{\theta}^2 = \frac{2b\sin^2\theta}{\cos^3\theta}\dot{\theta}^2$$

$$= 2b\tan^2\theta\sec\theta\,\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2\frac{b\sin\theta}{\cos^2\theta}\dot{\theta}^2 = 2b\tan\theta\sec\theta\,\dot{\theta}^2$$

(*a*) Radial and transverse components of effective forces.

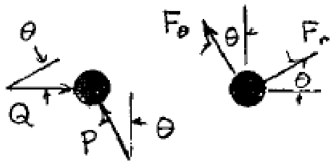
$$F_r = \frac{W}{g}a_r: \quad F_r = \frac{2Wb}{g}\tan^2\theta\sec\theta\,\dot{\theta}^2$$

$= 2(0.11)(0.225)10\text{^}2\tan^2\theta\sec\theta \qquad F_r = 4.95\tan^2\theta\sec\theta$ lb ◀

$$F_\theta = \frac{W}{g}a_\theta: \quad F_\theta = \frac{2Wb}{g}\tan\theta\sec\theta\,\dot{\theta}$$

$= 2(0.11)(0.225)10\text{^}2\tan\theta\sec\theta \qquad F_\theta = 4.95\tan\theta\sec\theta$ lb ◀

(*b*) Forces **P** and **Q** exerted on the pin by the arm *OA* and the wall of the slot *DE*, respectively.



$$\Sigma F_y = P\cos\theta = F_r\sin\theta - F_\theta\cos\theta$$

$\mathbf{P} = 4.95\tan\theta\sec^3\theta$ lb $\measuredangle\,\theta$ ◀

$$+\nearrow \Sigma F_r = Q\cos\theta = F_r$$

$\mathbf{Q} = 4.95\tan^2\theta\sec^2\theta$ lb $\longrightarrow$ ◀

SOLUTION PROBLEM 12.88

Circular orbits: $v = \sqrt{\frac{GM}{r}}$

$$r_A = 2240 \text{ km} = 2.24*10^{\wedge}6 \text{ m}$$

$$(v_A)_1 = \sqrt{\frac{(6.67*10^{\wedge}-11)(73.5*10^{\wedge}21)}{2.24*10^{\wedge}6}} = 1.479*10^{\wedge}3 \text{ m/s}$$

$$r_B = 2080 \text{ km} = 2.08*10^{\wedge}6 \text{ m}$$

$$(v_B)_2 = \sqrt{\frac{(6.67*10^{\wedge}-11)(73.5*10^{\wedge}21)}{2.08*10^{\wedge}6}} = 1.536*10^{\wedge}3 \text{ m/s}$$

(*a*) Transfer orbit *AB*.

$$(v_A)_2 = (v_A)_1 + (\Delta v)_A = 1.479*10^{\wedge}3 - 26.3 = 1.453*10^{\wedge}3 \text{ m/s}$$

$$mr_A(v_A)_2 = mr_B(v_B)_1$$

$$(v_B)_1 = \frac{r_A(v_A)_2}{r_B} = \frac{(1.479*10^{\wedge}3)(1.453*10^{\wedge}3)}{2.08*10^{\wedge}6} = 1.564*10^{\wedge}3 \text{ m/s}$$

$$(v_B)_1 = 1.564*10^{\wedge}3 \text{ m/s} \blacktriangleleft$$

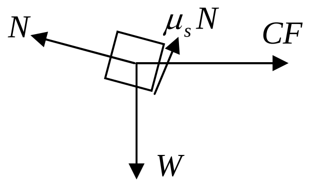
(*b*) Speed change at *B*.

$$(\Delta v_B) = (v_B)_2 - (v_B)_1 = 1.536*10^{\wedge}3 - 1.564*10^{\wedge}3 = -29.6 \text{ m/s}$$

Speed reduction at *B*. $|\Delta v_B| = 29.6 \text{ m/s} \blacktriangleleft$

12.127 Free body diagram for collar C is shown below, where the impending motion is assumed downward. This is the reason why the friction force directs upward.

$$CF : m\rho\omega^2 = 0.2 \cdot 0.6 \sin\theta \cdot 6^2$$



$$W \sin\theta - 0.2 \cdot 0.6 \sin\theta \cdot 6^2 \cos\theta - \mu_s N = 0 \quad (1)$$

$$N - W\cos\theta - 0.2 \cdot 0.6 \sin\theta \cdot 6^2 \sin\theta = 0 \quad (2)$$

(2) → $N = W\cos\theta + 0.2 \cdot 0.6 \sin^2\theta \cdot 6^2$ (2)'

(2)' → (1): $\mu_s = \dfrac{W \sin\theta - 0.2 \cdot 0.6 \sin\theta \cdot 6^2 \cos\theta}{W\cos\theta + 0.2 \cdot 0.6 \sin^2\theta \cdot 6^2}$

(a) $\theta = 90°, \mu_s = 0.454$

Impending motion is downward as assumed.

(b) $\theta = 75°, \mu_s = 0.1796$

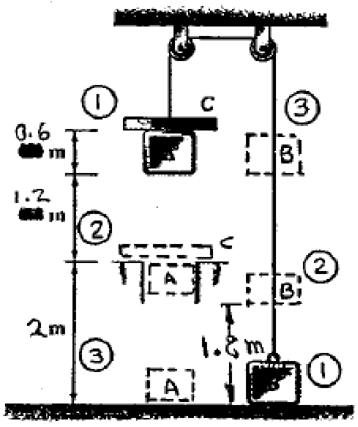
Impending motion is downward as assumed.

(c) $\theta = 45°, \mu_s = -0.2178$

In this case, the friction force acts downward, opposite to the direction assumed. Therefore, the impending motion is upward.

SOLUTION

PROBLEM 13.24



Given: $m_A = 8\text{ kg}$; $m_B = 10\text{ kg}$; $m_C = 6\text{ kg}$

System released from rest.

Collar C removed after blocks move 1.8 m.

Find: v_A, just before it strikes the ground.

Position 1 to position 2

$$v_1 = 0 \qquad T_1 = 0$$

At 2, before C is removed from the system

$$T_2 = \frac{1}{2}(m_A + m_B + m_C)v_2^2$$

$$T_2 = \frac{1}{2}(24\text{ kg})v_2^2 = 12v_2^2$$

$$U_{1-2} = (m_A + m_C - m_B)g(1.8\text{ m})$$

$$U_{1-2} = (8 + 6 - 10)g(1.8\text{ m}) = 70.632\text{ J}$$

$$T_1 + U_{1-2} = T_2; \qquad 0 + 70.632 = 12v_2^2$$

$$v_2^2 = 5.886$$

Position 2 to position 3

$$T_2' = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{18}{2}(5.886) = 52.974$$

$$T_3 = \frac{1}{2}(m_A + m_B)v_3^2 = 9v_3^2$$

$$U_{2'-3} = (m_A - m_B)g(2 - 0.6) = (-2\text{ kg})(9.81\text{ m/s}^2)(1.4\text{ m})$$

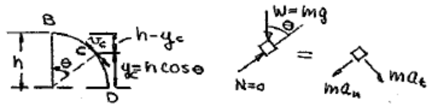
$$U_{2'-3} = -27.468\text{ J}$$

$$T_2' + U_{2'-3} = T_3 = 52.974 - 27.468 = 9v_3^2$$

$$v_3^2 = 2.834 \qquad v_3 = 1.68345 \qquad v_A = 1.683\text{ m/s} \blacktriangleleft$$

SOLUTION PROBLEM 13.45

(*a*)



Block leaves surface at C when the normal force $N = 0$

$$+\swarrow\ mg\cos\theta = ma_n$$

$$g\cos\theta = \frac{v_C^2}{h} \tag{1}$$

$$v_C^2 = gh\cos\theta = gy$$

Work-energy principle

$$T_B = \frac{1}{2}mv_C^2 \qquad U_{B-C} = W(h-y) = mg(h-y_C)$$

$$T_B + U_{B-C} = T_C$$

Use Equation (1)

$$4.5m + mg(h-y) = \frac{1}{2}mv_C^2$$

$$4.5 + g(h-y) = \frac{1}{2}gy_C \tag{2}$$

$$4.5 + gh = \frac{3}{2}gy_C$$

$$y_C = \frac{(4.5+gh)}{\left(\frac{3}{2}g\right)}$$

$$y = \frac{\left(4.5+(9.81)(1)^2\right)}{\frac{3}{2}(9.81)}$$

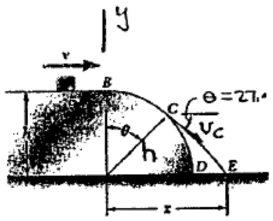
$$y = 0.97248\ \text{m} \tag{3}$$

$$y_C = h\cos\theta \qquad \cos\theta = \frac{y_C}{h} = \frac{0.97248}{1\ \text{m}} = 0.97248$$

$$\theta = \cos^{-1}0.97248 = 13.473°$$

$\theta = 13.47°$ ◀

(*b*)



From Equations (1) and (3)

$$v_C = \sqrt{gy} = \sqrt{9.81(0.97248)} = 3.0887\ \text{m/s}$$

At C;

$$(v_C)_x = v_C\cos\theta = 3.0887\cos 13.47°$$

$$= 3.0037\ \text{m/s}$$

$$(v_C)_y = -v_C\sin\theta = 3.0887\sin 13.47°$$

$$= -0.71947\ \text{m/s}$$

$$y = y_C + (v_C)_y t - \frac{1}{2}gt^2 = 0.97248 - 0.71947t - \frac{1}{2}(9.81)t^2$$

At E:

$$y_E = 0:\quad 4.905t^2 + 0.7194t - 0.97248 = 0$$

$$t = 0.37793\ \text{s}$$

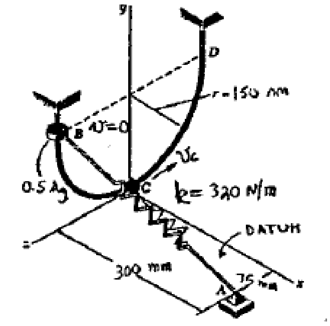
At E:

$$x = h\cos\theta + (v_C)_x t = 1(\sin 13.47°) + 3.0037(0.37793)$$

$$= 0.23294 + 1.3519 = 1.3681\ \text{m}$$

$x = 1.368$ m ◀

SOLUTION PROBLEM 13.69



(*a*) Speed at C

$$L_{AB} = \sqrt{(300)^2 + (150)^2 + (75)^2} = 343.69318 \text{ mm}$$

$$k = 320 \text{ N/m}$$

At B $\quad v_B = 0 \qquad T_B = 0$

$$V_B = (V_B)_e + (V_B)_g$$

$$\Delta L_{AB} = 343.69318 \text{ mm} - 200 \text{ mm}$$

$$\Delta L_{AB} = 143.69318 \text{ mm} = 0.14369318 \text{ m}$$

$$(V_B)_e = \frac{1}{2}k(\Delta L_{AB})^2 = \frac{1}{2}(320 \text{ N/m})(0.1436932 \text{ m})^2$$

$$(V_B)_e = 3.303637 \text{ J}$$

$$(V_B)_g = Wr = (0.5 \text{ kg})(9.81 \text{ m/s}^2)(0.15 \text{ m}) = 0.73575 \text{ J}$$

$$V_B = (V_B)_e + (V_B)_g = 3.303637 \text{ J} + 0.73575 \text{ J} = 4.03939 \text{ J}$$

At C $\quad T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.5 \text{ kg})(v_C^2)$

$$T_C = 0.25v_C^2$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{AC})^2$$

$$\Delta L_{AC} = 309.23 \text{ mm} - 200 \text{ mm} = 109.23 \text{ mm} = 0.10923 \text{ m}$$

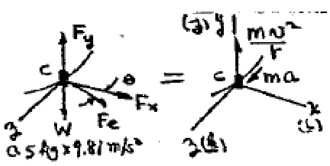
$$(V_C)_e = \frac{1}{2}(320 \text{ N/m})(0.10923 \text{ m})^2 = 1.90909 \text{ J}$$

$$T_B + V_B = T_C + V_C$$

$$0 + 4.0394 = 0.25v_C^2 + 1.90909$$

$$v_C^2 = \frac{4.0394 - 1.90909}{0.25} = 8.5212 \text{ m}^2/\text{s}^2 \qquad v_C = 2.92 \text{ m/s} \blacktriangleleft$$

(*b*) Force of rod on collar *AC*



$$F_z = 0 \quad \text{(no friction)}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

$$\theta = \tan^{-1}\frac{75}{300} = 14.04°$$

$$\mathbf{F}_e = (k\Delta L_{AC})(\cos\theta\mathbf{i} + \sin\theta\mathbf{k})$$

$$\mathbf{F}_e = (320)(0.10923)(\cos 14.04°\mathbf{i} + \sin 14.04°\mathbf{k})$$

$$\mathbf{F}_e = 33.909\mathbf{i} + 8.4797\mathbf{k} \text{ (N)}$$

$$\Sigma\mathbf{F} = (F_x + 33.909)\mathbf{i} + (F_y - 4.905)\mathbf{j} + 8.4797\mathbf{k} = \frac{mv^2}{r}\mathbf{j} + mg\mathbf{k}$$

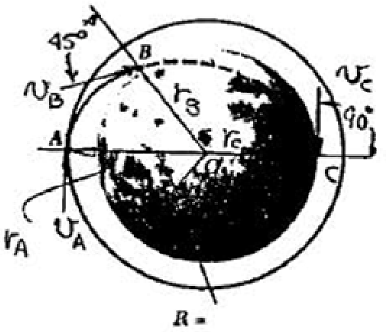
$$F_x + 33.909 \text{ N} = 0 \qquad F_y = 4.905 \text{ N} + (0.5)\frac{(8.5212 \text{ m}^2/\text{s}^2)}{0.15 \text{ m}}$$

$$F_x = -33.909 \text{ N}$$

$$F_y = 33.309 \text{ N}$$

$$\mathbf{F} = -33.9 \text{ N}\,\mathbf{i} + 33.3 \text{ N}\,\mathbf{j} \blacktriangleleft$$

SOLUTION   PROBLEM 13.112



$$r_A = 1740 + 140 = 1880 = 1.88*10^6 \text{ m}$$

$$r_C = R = 1740 = 1.74*10^6 \text{ m}$$

$$GM_{moon} = 0.0123 \qquad GM_E = 0.0123 g R_E^2$$

$$= 0.0123(9.8)(6.37*10^6)^2$$

$$= 4.89*10^{12} \text{ m}^3/\text{s}^2$$

At 87 mi: $$v_{circ} = \sqrt{\frac{GM_{moon}}{r_A}} = 1613 \text{ m/s}$$

(*a*) An elliptic trajectory between *A* and *C*, where the lem is just tangent to the surface of the moon, will give the smallest reduction of speed at *A* which will cause impact.

$$T_A = \frac{1}{2}mv_A^2 \qquad V_A = -\frac{GM_m m}{r_A} = -2.60*10^6 \text{ m}$$

$$T_C = \frac{1}{2}mv_C^2 \qquad V_C = -\frac{GM_m m}{r_C} = -2.81*10^6 \text{ m}$$

$$T_A + V_A = T_C + V_C: \quad \frac{1}{2}mv_A^2 \; -2.60*10^6 \text{ m}$$

$$= \frac{1}{2}mv_C^2 \; -2.81*10^6 \text{ m}$$

$$v_A^2 = v_C^2 - 4.2*10^5 \qquad (1)$$

Conservation of angular momentum: $\quad r_A m v_A = r_C m v_C$

$$v_C = \frac{r_A}{r_C}v_A = \frac{1.88*10^6}{1.74*10^6}v_A = 1.0806v_A$$

$$v_A^2 = (1.0806v_A)^2 - 4.2*10^5 \Rightarrow v_A = 1582.57 \text{ m/s}$$

$$\Delta v_A = (v_A)_{circ} - v_A = 1613 - 1582.57 = 30.34$$

$\Delta v_A = 30.34$ m/s ◀

(*b*) Conservation of energy (A and B)

Since $r_B = r_C$ conservation of energy is the same as between *A* and *C*

Conservation of angular momentum:

$$r_A m v_A = r_B m v_B \sin\phi; \qquad \phi = 45°$$

$$v_B = \frac{r_A}{r_B}\frac{v_A}{\sin 45°} = \left(\frac{1.88*10^6}{1.74*10^6}\right)\left(\frac{V_A}{0.70711}\right) = 1.5281v_A$$

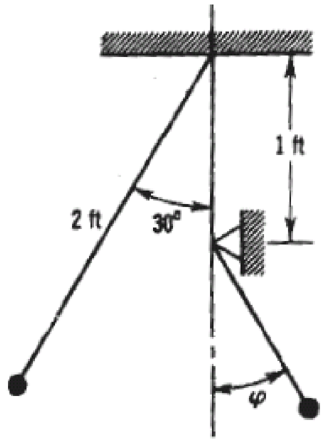
From (1)

$$v_A^2 = (1.5281v_A)^2 - 4.2*10^5 \Rightarrow v_A = 560 \text{ ft/s}$$

$$\Delta v_A = (v_A)_{circ} - v_A = 1613 - 560 = 1053$$

$\Delta v_A = 1053$ m/s ◀

1



(a) $1 + 1 \cdot \cos\phi = 2\cos 30 = \sqrt{3}$

$\cos\phi = \sqrt{3} - 1 \qquad \therefore \phi = 42.94^\circ = 42^\circ 57'$

(b) when $\phi = 30^\circ$

$F = m\frac{v^2}{r} + mg\cos\phi$

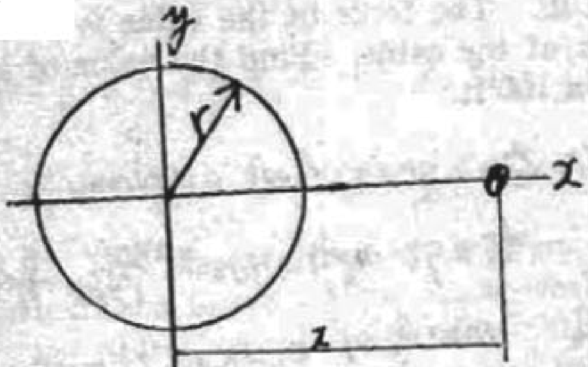
here $mgl(1-\cos 30) = mg\frac{l}{2}(1-\cos\phi) + \frac{1}{2}mv^2$

$32.2 \times 2\left(1-\frac{\sqrt{3}}{2}\right) = 32.2 \times 1\left(1-\frac{\sqrt{3}}{2}\right) + \frac{v^2}{2}$

$\therefore v = 2.937$ ft/sec

$\therefore F = m\frac{v^2}{r} + mg\cos 30$

$= \frac{1 \times (2.937)^2}{32.2 \times 1} + 1 \cdot \frac{\sqrt{3}}{2} = 1.134\ (lb)$



gravitational force

$$F_x = -W\frac{r^2}{x^2}$$

① $\therefore work = \int_r^x F_x\,dx = \int_r^x -W\frac{r^2}{x^2}\,dx = W\frac{r^2}{x}\Big|_r^x$

$$= Wr^2\left(\frac{1}{x}-\frac{1}{r}\right)$$

Work_against $= -Wr^2\left(\frac{1}{x}-\frac{1}{r}\right)$

② $-\frac{1}{2}mv^2 = wr^2\left(-\frac{1}{x}+\frac{1}{r}\right)$

$$v = \sqrt{2gr^2\left(-\frac{1}{x}+\frac{1}{r}\right)}$$

③ 지구로 부터 이탈하기 위해서는 $x = \infty$

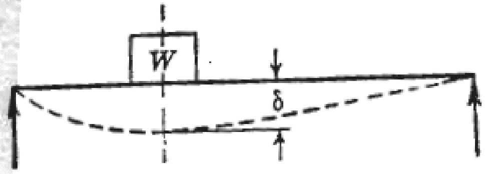
$$\therefore \frac{1}{2}\frac{W}{g}v^2 = Wr$$

or from ② $\therefore v = \sqrt{2gr}$

$$= \sqrt{2\times32.2\times5280\times4000}$$

$$= 36900 \text{ ft/sec}$$

the beam.



$$W(h+\delta) - \int_0^{\delta} kx\,dx = 0$$

$$W(h+\delta) = \frac{1}{2} k \delta^2 \quad (\text{here, } k = \frac{W}{\delta_{st}})$$

$$\frac{\delta^2}{2\delta_{st}} = (h+\delta)$$

$$\delta^2 - 2\delta_{st}\delta - 2h\delta_{st} = 0$$

$$\delta = \delta_{st} + \sqrt{\delta_{st}^2 + 2h\delta_{st}} = \delta_{st}\lbrack 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$