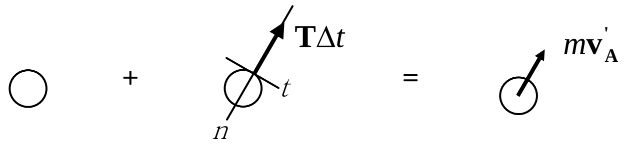
동역학 446.204A 003

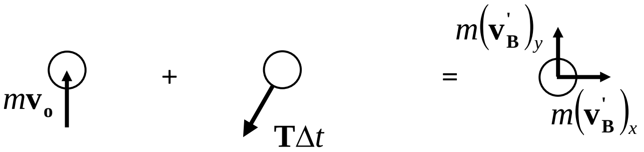
과제물 #4 모범 답안

13.149 Apply the principle of impulse and momentum to each sphere one by one.
For sphere A, consider the direction in line of impact.



$$T\Delta t = mv_A' \tag{1}$$

Therefore, sphere A will move in the direction of line of impact.
For sphere B, apply the principle of impulse and momentum to the horizontal and vertical direction.



$$\rightarrow: 0 - T\Delta t \cdot \frac{a}{L} = m\left(v_B'\right)_x \tag{2}$$

$$\uparrow: mv_o - T\Delta t \cdot \frac{\sqrt{L^2 - a^2}}{L} = m\left(v_B'\right)_y \tag{3}$$

(1) ⟶ (2): $$-v_A' \cdot \frac{a}{L} = \left(v_B'\right)_x \tag{4}$$

(1) ⟶ (3): $$v_o - v_A' \cdot \frac{\sqrt{L^2 - a^2}}{L} = \left(v_B'\right)_y \tag{5}$$

Since the cord is inextensible and inelastic, the impact can be regarded perfectly plastic (e = 0). This is as if the cord was replaced with a rigid link at the moment immediately after the impact. From the expression for the coefficient of restitution,

$$\left(v_B'\right)_n - \left(v_A'\right)_n = e\left\{\left(v_A\right)_n - \left(v_B\right)_n\right\} = 0 \tag{6}$$

(6) ⟶ : $$v_A' = \left(v_B'\right)_n = \left(v_B'\right)_x \cos\theta + \left(v_B'\right)_y \sin\theta = \left(v_B'\right)_x \frac{a}{L} + \left(v_B'\right)_y \frac{\sqrt{L^2 - a^2}}{L} \tag{6)'}$$

(4), (5) ⟶ (6)': $$v_A' = -v_A'\left(\frac{a}{L}\right)^2 + \left(v_o - v_A' \frac{\sqrt{L^2 - a^2}}{L}\right)\frac{\sqrt{L^2 - a^2}}{L}$$

$$v_A'\left\{1+\left(\frac{a}{L}\right)^2+\frac{L^2-a^2}{L^2}\right\}=v_o\frac{\sqrt{L^2-a^2}}{L}$$

$$v_A'=\frac{v_o}{2L}\sqrt{L^2-a^2} \qquad (7)$$

(7) ⟶ (4): $\left(v_B'\right)_x=-v_A'\cdot\frac{a}{L}=-v_A'\cdot\frac{a}{2L^2}\sqrt{L^2-a^2}$

(7) ⟶ (5): $\left(v_B'\right)_y=v_o-v_A'\cdot\frac{\sqrt{L^2-a^2}}{L}=v_o-v_o\cdot\frac{1}{2L}\sqrt{L^2-a^2}\cdot\frac{\sqrt{L^2-a^2}}{L}$

$$\left(v_B'\right)_y=v_o\frac{L^2+a^2}{2L^2}$$

$$v_B'=\sqrt{\left(v_B'\right)_x^2+\left(v_B'\right)_y^2}=v_o\sqrt{\left(\frac{a}{2L^2}\sqrt{L^2-a^2}\right)^2+\left(\frac{L^2+a^2}{2L^2}\right)^2}$$

$$v_B'=\frac{v_o}{2L}\sqrt{L^2+3a^2}$$

(b) Regarding the loss of energy, we need to consider only kinetic energy of the two spheres since there are no changes in potential energy due to gravity force.

$$T_1=\frac{1}{2}mv_o^2$$

$$T_2=\frac{1}{2}m\left\{\left(v_A'\right)^2+\left(v_B'\right)^2\right\}=\frac{1}{2}mv_o^2\frac{1}{2L^2}\left(L^2+a^2\right)$$

$$\Delta T=T_1-T_2=\frac{1}{2}mv_o^2\left\{1-\frac{1}{2L^2}\left(L^2+a^2\right)\right\}=\frac{mv_o^2}{4L^2}\left(L^2-a^2\right)$$

13.179 (a) For the impact between the carrier A and B which happens first, apply the conservation of linear momentum and the expression of the coefficient of restitution.

$$40\cdot v_A=40\cdot v_A'+40\cdot v_B' \qquad (1)$$

$$v_B'-v_A'=0.8\cdot\left(v_A-v_B\right) \qquad (2)$$

(2) ⟶ : $v_B'=v_A'+0.8\cdot v_A$ (2)'

(2)' ⟶ (1): $40\cdot5=40\cdot v_A'+40\cdot v_A'+40\cdot0.8\cdot5$

$$80\cdot v_A'=200(1-0.8)$$

$$v_A'=0.5(\mathrm{m/s}), v_B'=4.5(\mathrm{m/s})$$

Then, apply the same procedures for the impact between the carrier B and the suitcase C which happens next,

$$40 \cdot v_B' = 40 \cdot v_B'' + 15 \cdot v_C'' \qquad (3)$$

$$v_B'' - v_C'' = 0.3 \cdot \left(v_C' - v_B' \right) \qquad (4)$$

(4) ⟶ : $$v_B'' = v_C'' - 0.3 \cdot v_B' \qquad (4)'$$

(4)' ⟶ (3): $$40 \cdot 5 = 40 \cdot v_C'' + 15 \cdot v_C'' - 40 \cdot 0.3 \cdot 4.5$$

$$55 \cdot v_C'' = 40 \cdot 4.5(1 + 0.3)$$

$$v_B'' = 4.25(\mathrm{m/s}), v_C'' = 2.90(\mathrm{m/s})$$

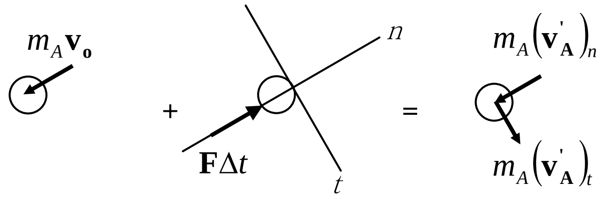
(b) Considering only the kinetic energy of the two carriers A, B and the suitcase C between the initial state (before the impact between B and C) and the final state (after the impact between B and C), since there are no change in potential energy.

$$T_1 = \frac{1}{2} \cdot 40 \cdot \left(0.5^2 + 4.5^2 \right)$$

$$T_2 = \frac{1}{2} \cdot 40 \cdot \left(0.5^2 + 2.90^2 \right) + \frac{1}{2} \cdot 15 \cdot 4.25^2$$

$$T_1 - T_2 = 101.3(\mathrm{J})$$

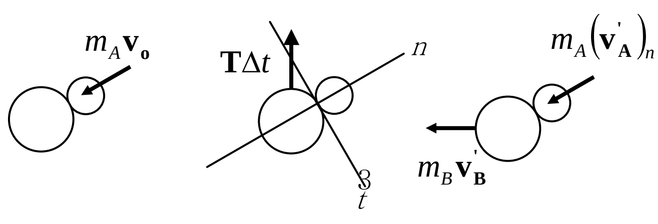
13.189 First, considering the ball A only, apply the principle of impulse and momentum to the t-component,



$$\left(v_A' \right)_t = 0$$

Therefore, after the impact, the ball B has only component of its velocity along the line of impact (n-axis).

Now considering the two balls together, apply the principle of impulse and momentum to the horizontal component,



+ =

$$-m_A v_o \sin 60° = -m_A \left(v_A^{'}\right)_n \sin 60° - m_B v_B^{'} \qquad (1)$$

From the expression of the coefficient of restitution,

$$\left(v_B^{'}\right)_n - \left(v_A^{'}\right)_n = \left(v_A\right)_n - \left(v_B\right)_n$$

$$v_B^{'} \sin 60° - \left(v_A^{'}\right)_n = v_o - 0 \qquad (2)$$

Therefore, after the impact, the ball B has only component of its velocity along the line of impact (n-axis).

(2) $\longrightarrow$: $\left(v_A^{'}\right)_n = v_B^{'} \sin 60° - v_o$ (2)'

(2)' $\longrightarrow$ (1): $-m_A v_o \sin 60° = -m_A \left(v_B^{'} \sin 60° - v_o\right)_n \sin 60° - m_B v_B^{'}$

$$\left(-m_A v_o \sin^2 60° - m_B\right) v_B^{'} = -m_A v_o \sin 60° - m_A v_o \sin 60°$$

$$v_B^{'} = \frac{-2 m_A v_o \sin 60°}{-m_A v_o \sin^2 60° - m_B} = 0.945 \left(\mathrm{m/s}\right)$$

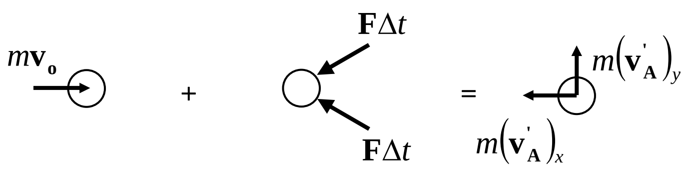
To obtain the height reached by the ball B, apply the principle of conservation of energy to the ball B.

$$\frac{1}{2} m_B \left(v_B^{'}\right)^2 = m_B g h$$

$$h = \frac{\left(v_B^{'}\right)^2}{2g} = 0.0455 \left(\mathrm{m}\right)$$

14.38 (a) First apply the principle of impulse and momentum to each ball.

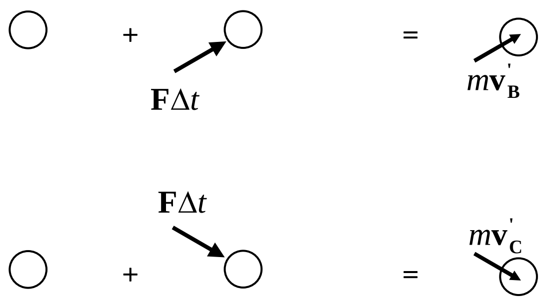
Ball A is acted upon the same magnitude of impulses by the two balls B and C in the impact, as shown. Consider only the vertical component of impulse and momentum.

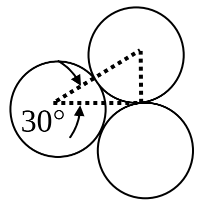


$$0 + F_y \Delta t - F_y \Delta t = m\left(v_A^{'}\right)_y$$

$$\left(v_A^{'}\right)_y = 0$$

Therefore, Ball A will recoil in the horizontal direction.
For Ball B and C, their impulse-momentum diagrams are symmetric with each other.





$$\left(v_B^{'}\right)_x = \left(v_C^{'}\right)_x, \left(v_B^{'}\right)_y = \left(v_C^{'}\right)_y$$

$$\left(v_B^{'}\right)_x = v_B^{'} \cos 30°, \left(v_B^{'}\right)_x = v_B^{'} \sin 30°$$

Regarding the assumption of perfectly elastic impact, we need to consider only the kinetic energy of the balls since there are no changes in potential energy.

$$\frac{1}{2} m v_o^2 = \frac{1}{2} m \left(v_A^{'}\right)_x^2 + m\left(\left(v_B^{'}\right)_x^2 + \left(v_B^{'}\right)_y^2\right) \tag{1}$$

Then, apply the principle of impulse and momentum to the all balls together, and consider the horizontal component.

$$m v_o = m v_A^{'} + 2m\left(v_B^{'}\right)_x = m v_A^{'} + 2 m v_B^{'} \cos 30° \tag{2}$$

(2) $\longrightarrow$: $\quad v_A^{'} = v_o - 2 v_B^{'} \cos 30°$ (2)'

(2)' $\longrightarrow$ (1): $v_o^2 = \left(v_A^{'}\right)_x^2 + 2\left(v_B^{'}\right)^2 \left(\sin^2 30° + \cos^2 30°\right)$

$$= \left(v_A^{'}\right)_x^2 + 2\left(v_B^{'}\right)^2 = v_o^2 - 4 v_o v_B^{'} \cos 30° + 4\left(v_B^{'}\right)^2 \cos^2 30° + 2\left(v_B^{'}\right)^2$$

$$\left(2 + 4\cos^2 30°\right)\left(v_B^{'}\right)^2 - 4 v_o \cos 30° v_B^{'} = 0$$

$$v_B' = \frac{4v_o \cos 30°}{2 + 4\cos^2 30°} = 0.693 v_o$$

$$v_A' = (1 - 2\cos 30° \cdot 0.693) v_o = -0.2 v_o$$

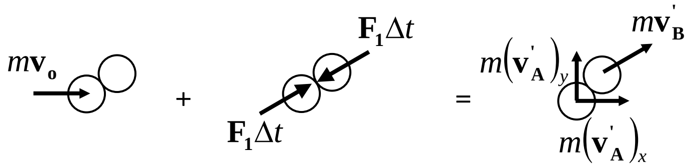
$$v_A' = 0.2 v_o \leftarrow$$

$$v_B' = 0.693 v_o \qquad 30°$$

$$v_C' = 0.693 v_o \qquad 30°$$

(b) Now consider the two impacts sequentially.

For the first impact between ball A and B, apply the principle of impulse and momentum to the two balls.



$$\rightarrow: mv_o = m(v_A')_x + m(v_B')_x = m(v_A')_x + mv_B' \cos 30° \tag{1}$$

$$\uparrow: 0 = m(v_A')_y + m(v_B')_y = m(v_A')_y + mv_B' \sin 30° \tag{2}$$

It is noted that the ball B will move in the direction of line of impact by applying the impulse and momentum principle to the ball B only.

From the assumption of the perfectly elastic impact,

$$\frac{1}{2}mv_o^2 = \frac{1}{2}m(v_A')^2 + \frac{1}{2}m(v_B')^2 = \frac{1}{2}m\left((v_A')_x^2 + (v_A')_y^2\right) + \frac{1}{2}m(v_B')^2 \tag{3}$$

(1) $\longrightarrow$: $\quad (v_A')_x = v_o - v_B' \cos 30°$ (1)'

(2) $\longrightarrow$: $\quad (v_A')_y = v_B' \sin 30°$ (2)'

(1)',(2)' $\longrightarrow$ (3): $\quad v_o^2 = (v_o - v_B' \cos 30°)^2 + (v_B')^2 \sin^2 30° + (v_B')^2$

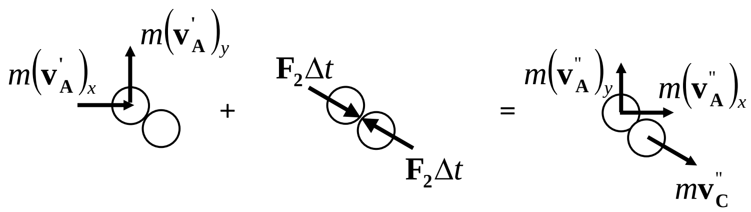
$$v_B'(2v_B' - 2\cos^2 30° v_o) = 0$$

$$v_B' = \cos 30° v_o = 0.866 v_o$$

$$(v_A')_x = v_o - \cos^2 30° v_o = 0.25 v_o$$

$$(v_A')_y = -\sin 30° \cos 30° v_o = -0.433 v_o$$

Considering the second impact between the ball B and C, apply the same procedures as used above.



$$\rightarrow: m \cdot 0.25 \cdot v_o = m(v_A'')_x + m(v_C'')_x = m(v_A'')_x + m v_C'' \cos 30° \quad (4)$$

$$\uparrow: -m \cdot 0.433 v_o = m(v_A'')_y + m(v_C'')_y = m(v_A'')_y - m v_C'' \sin 30° \quad (5)$$

It is noted that the ball C will move in the direction of line of impact by applying the impulse and momentum principle to the ball C only.
From the assumption of the perfectly elastic impact,

$$\frac{1}{2} m (0.25^2 + 0.433^2) v_o^2 = \frac{1}{2} m (v_A'')^2 + \frac{1}{2} m (v_C'')^2 = \frac{1}{2} m \left((v_A'')_x^2 + (v_A'')_y^2\right) + \frac{1}{2} m (v_C'')^2 \quad (6)$$

(4) $\longrightarrow$: $(v_A'')_x = 0.25 v_o - v_C'' \cos 30°$ (4)'

(5) $\longrightarrow$: $(v_A'')_y = -0.433 v_o + v_C'' \sin 30°$ (5)'

(5)',(6)' $\longrightarrow$ (4):

$$(0.25^2 + 0.433^2) v_o^2 = (0.25 v_o - v_C'' \cos 30°)^2 + (-0.433 v_o + v_C'' \sin 30°)^2 \sin^2 30° + (v_C'')^2$$

$$2 v_C'' \left\{ v_C'' + (-2 \cdot 0.433 \cdot \sin 30° - 2 \cdot 0.25 \cdot \cos 30°) v_o \right\} = 0$$

$$v_C'' = (0.25 \cos 30° + 0.433 \sin 30°) v_o = 0.433 v_o$$

$$(v_A'')_x = (0.25 - 0.433 \cos 30°) v_o = -0.125 v_o$$

$$\left(v_A''\right)_y = \left(-0.433 + 0.433\sin 30°\right)v_o = -0.2165v_o$$

$v_A'' = 0.25v_o$ 60°

$v_B' = 0.866v_o$ 30°

$v_C' = 0.433v_o$ 30°

14. 44 (a) Let us analyze the impact between *B* and *C*, which happens first.

Sphere *B* and *C* are acted upon by the action/reaction impulses in the vertical direction, so they will move in the vertical direction after the impact.

Apply the principle of impulse and momentum in the vertical direction.

$$\downarrow: m_A v_o = m_A v_B' + m_B v_C' \tag{1}$$

From the expression of the coefficient of restitution,

$$\left(v_C' - v_B'\right) = e\left(v_B - v_C\right) = -v_o \tag{2}$$

(2) ⟶ : $v_C' = v_B' - v_o$ (2)'

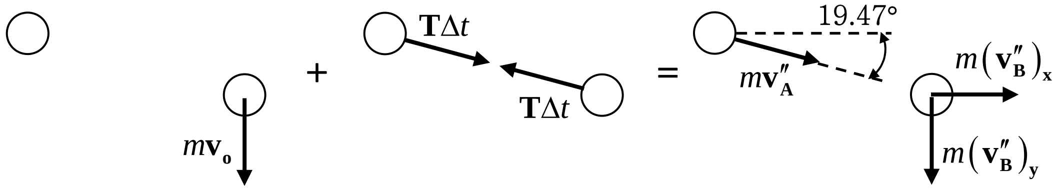
(2)'⟶ (1): $v_o = v_B' - v_o + v_B'$

$2v_o = 2v_B'$

$v_B' = v_o \rightarrow, v_C' = 0$

Therefore, after the first impact, sphere *C* halts, and sphere *B* will move in the downward direction with $\mathbf{v_o}$.

Then, let us analyze the second impact between *A* and *B*, immediately after the cord becomes taut. Sphere *A* will move in the direction of the cord immediately after the cord becomes taut, since the impulse due to the tension in the cord acts in that direction. However, the direction in which sphere *B* will move immediately after the cord becomes taut is unknown, so its horizontal and vertical components are to be determined as shown below.



Apply the principle of impulse and momentum in the horizontal and vertical directions.

$$\downarrow: mv_o = m_A v_A'' \sin 19.47° + m_B \left(v_B''\right)_y \qquad (3)$$

$$\rightarrow: 0 = m_A v_A'' \cos 19.47° + m_B \left(v_B''\right)_x \qquad (4)$$

Since the cord is inextensible and inelastic, the impact can be regarded as perfectly plastic (e = 0). From the expression of the coefficient of restitution,

$$\left(v_B''\right)_n - \left(v_A''\right)_n = e\left\{\left(v_A'\right)_n - \left(v_B'\right)_n\right\} = 0 \qquad (5)$$

(4) $\longrightarrow$: $\left(v_B''\right)_x = -v_A'' \cos 19.47°$ (4)'

(5) $\longrightarrow$: $v_A'' = \left(v_B''\right)_n = \left(v_B''\right)_x \cos 19.47° + \left(v_B''\right)_y \sin 19.47°$

$$\left(v_B''\right)_y = v_A'' \frac{1}{\sin 19.47°} - \left(v_B''\right)_x \frac{\cos 19.47°}{\sin 19.47°} \qquad (5)'$$

(4)', (5)' $\longrightarrow$ (3): $v_o = v_A'' \sin 19.47° + \left(v_B''\right)_y$

$$= v_A'' \sin 19.47° + v_A'' \frac{1}{\sin 19.47°} - \left(v_B''\right)_x \frac{\cos 19.47°}{\sin 19.47°}$$

$$= v_A'' \sin 19.47° + v_A'' \frac{1}{\sin 19.47°} + v_A'' \frac{\cos^2 19.47°}{\sin 19.47°}$$

$$= v_A'' \left(\sin 19.47° + \frac{1}{\sin 19.47°} + \frac{\cos^2 19.47°}{\sin 19.47°} \right)$$

$$v_A'' = v_o \frac{1}{\left(\sin 19.47° + \dfrac{1}{\sin 19.47°} + \dfrac{\cos^2 19.47°}{\sin 19.47°} \right)} = 0.1667 v_o \qquad 19.47°$$

(4)' $\longrightarrow$: $\left(v_B''\right)_x = -0.1667 v_o \cdot \cos 19.47° = -0.1571 v_o$

(5)' $\longrightarrow$: $\left(v_B''\right)_y = 0.1667 v_o \dfrac{1}{\sin 19.47°} + 0.1571 v_o \dfrac{\cos 19.47°}{\sin 19.47°} = 0.9443 v_o$

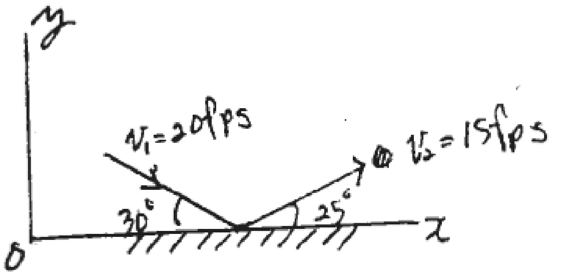
$$v_B'' = 0.9573 v_o \qquad 80.6°$$

(b) $T_1 = \frac{1}{2} m v_o^2$

$$T_2 = \frac{1}{2} m \left\{ \left(v_A''\right)^2 + \left(v_B''\right)^2 \right\} v_o^2 = \frac{1}{2} m \left(0.1667^2 + 0.9573^2\right) v_o^2 = \frac{1}{2} m \cdot 0.9442 v_o^2$$

$$\frac{\Delta T}{T_1} = \frac{T_1 - T_2}{T_1} = \frac{1 - 0.9442}{1} = 0.0558$$

3.9



$W = 5$ lb

$$\begin{cases} V_{1x} = 20 \cos 30 & V_{1y} = 20 \sin 30 \\ V_{2x} = 15 \cos 25 & V_{2y} = 15 \sin 25 \end{cases}$$

$$I_x = \int F_x \, dt = \frac{W}{g} (V_{2x} - V_{1x}) = \frac{5}{32.2} (15 \cos 25 - 20 \cos 30) = -0.57855$$

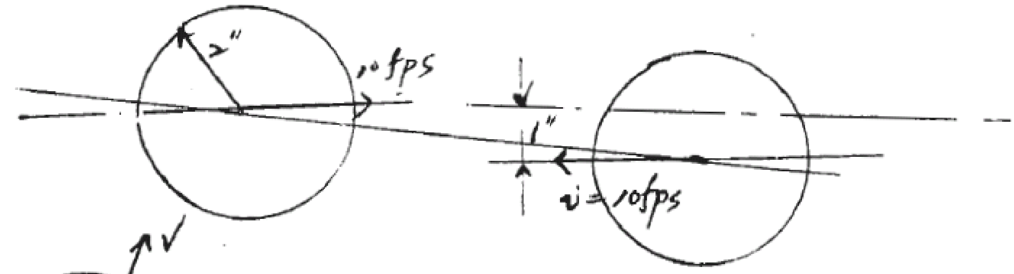
$$I_y = \int F_y \, dt = \frac{W}{g} (V_{2y} - V_{1y}) = \frac{5}{32.2} (15 \sin 25 + 20 \sin 30) = 2.537$$

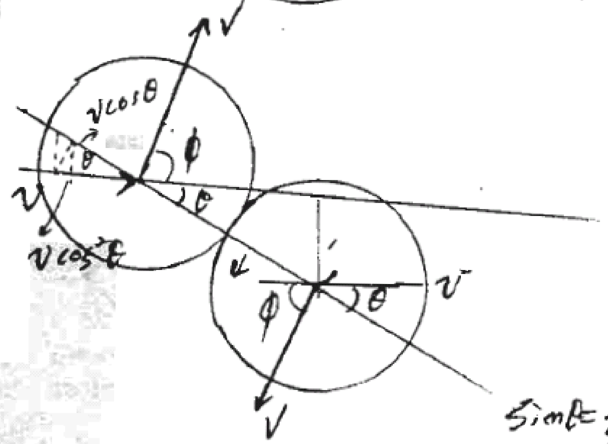
$$\therefore I = \sqrt{I_x^2 + I_y^2} = 2.602 \quad \text{lb}\cdot\text{sec}$$

$$\tan \phi = \frac{I_y}{I_x} = -\frac{2.537}{0.57855}$$

$$\therefore \phi = 102.85° = 102°50' \quad \text{or} \quad \phi = -77°10'$$

4.30





$$e = 5/9$$

$$e = -\frac{V\sin\phi - (-V\sin\phi)}{v\cos^2\theta - (-v\cos^2\theta)}$$

$$= -\frac{V\sin\phi}{v\cos^2\theta} = 5/9$$

$\sin\theta = \frac{1}{4}$ $\quad \cos\theta = \frac{\sqrt{15}}{4}$

$$mV\sin\phi = mv\cos\theta\cdot\sin\theta$$

$$V\sin\phi = v\cos\theta\cdot\sin\theta$$

$$V\cos\phi = -\frac{5}{9}v\cos^2\theta$$

$$\tan\phi = -\frac{5}{9}\tan\theta = -\frac{5}{9}\frac{1}{\sqrt{15}} = -0.465$$

$$\therefore \phi = 155^\circ$$

$$V = \frac{10\times\frac{\sqrt{15}}{4}\times\frac{1}{4}}{\sin\phi} = 5.73 \text{ ft/sec}$$

4.33

a) $mv + m0 = mv_1 + mv_2$

$v_1 - v_2 = -(v - 0)$

$$\begin{cases} v_1 + v_2 = v \\ v_1 - v_2 = -v \end{cases} \quad \therefore 2v_1 = 0 \quad \therefore v_1 = 0 \quad v_2 = v$$

∴ 마지막의 m이 $v(\rightarrow)$의 velocity를 갖는다

(b) If the distances d approach zero (zero = d) assume

$mv + 4m(0) = mv_1' + 4mv_2'$

$v_1' - v_2' = -(v - 0)$

$$\begin{cases} v_1' + 4v_2' = v \\ v_1' - v_2' = -v \end{cases} \quad 5v_2' = 2v \quad \therefore v_2' = \frac{2}{5}v \quad v_1' = -\frac{3}{5}v$$

∴ 4m의 m의 $\frac{2}{5}v(\rightarrow)$의 속도를 갖고

impact m은 $\frac{3}{5}v(\leftarrow)$의 속도를 갖는다.

6.4

(a) $\mu mg = ma_1 \quad \therefore a_1 = \mu g$

$\therefore F - F_1 = Ma_2$

$F - \mu mg = Ma_2 \quad \therefore a_2 = \frac{1}{M}(F - \mu mg)$

(b) $l = \frac{1}{2}a_{1/2}t^2$

here $a_{1/2} = \mu g - \frac{1}{M}(F - \mu mg)$

$\therefore S_M = \frac{1}{2} a_2 t^2$

위 식에서 $t^2 = \frac{2l}{a_{1/2}} = \frac{2l}{\mu g - \frac{F - \mu m g}{M}}$

$\therefore S_M = \frac{1}{2}\frac{1}{M}(F - \mu m g)\frac{2lM}{\mu M g - F + \mu m g}$

$= \frac{(F - \mu m g)l}{\mu(M+m)g - F}$

6.6

i) $r_c = \frac{bw + naw}{2(n+1)w} = \frac{(\frac{a}{2} - x)nw + (a - x - \frac{b}{2})w}{(n+1)w}$

OR $x = \frac{a-b}{n+1}$

ii) OR use the concept of 6-2

$S_1 + S_2 = a - b$

$2aS = a^2 t^2 \quad \therefore S \propto \frac{1}{a}$

$\therefore S_1 : S_2 = 1 : n$

hence $S_1 = \frac{a-b}{n+1}$