465.420 Geothermal Energy, Fall 2009

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Homework #2

1.

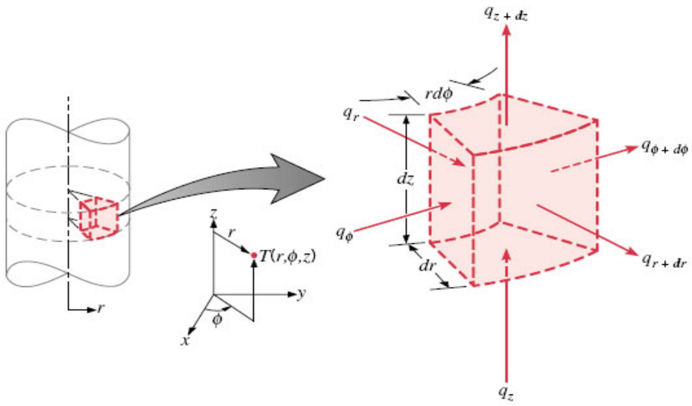
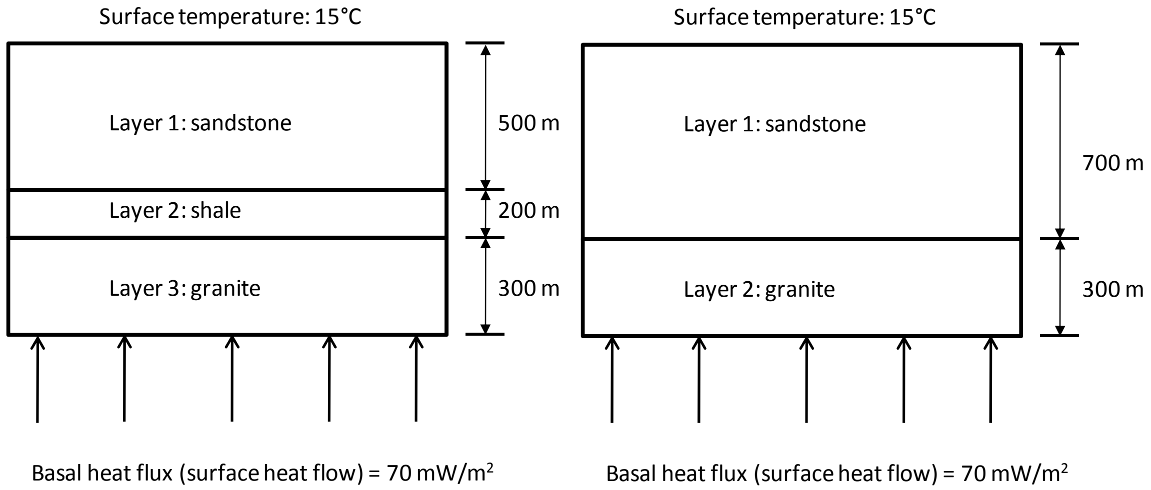


FIGURE 2.12 Differential control volume, $dr \cdot r\,d\phi \cdot dz$, for conduction analysis in cylindrical coordinates (r, ϕ, z).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

Derive the heat diffusion equation in cylindrical coordinate beginning with the differential control volume shown above.

2.



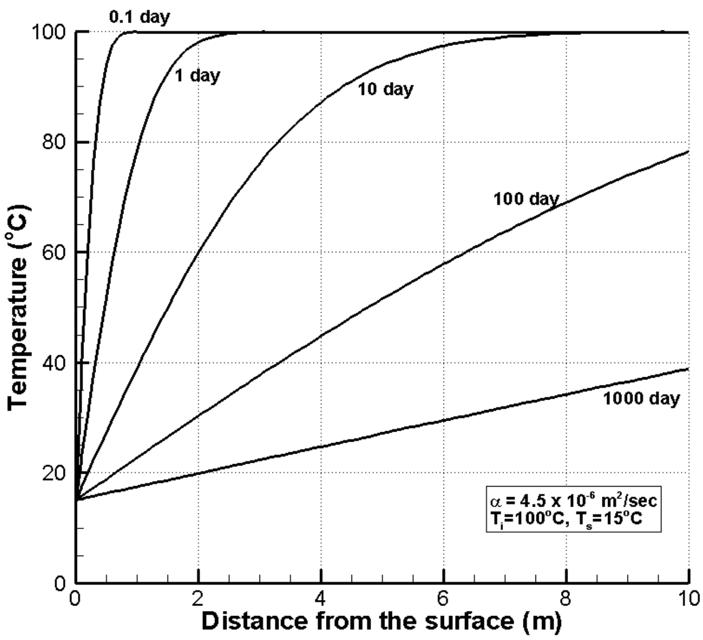
A particular sequence of rocks, 1 km thick, is composed of horizontally layered, interbedded sandstone, shale and granite. The surface temperature is 15°C and basal flux is 70 W/m^2. Thermal conductivity of sandstone : 2.7 W/mK, Thermal conductivity of shale: 1.7 W/mK, thermal conductivity of granite: 3.5 W/mK.

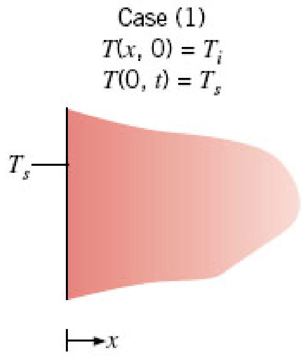
Assuming that there is no heat source within the layers and lateral boundaries are insulated, estimate the temperature at the depth of 1 km and calculate the thermal gradient of each layer, overall gradient of 1 km depth and equivalent thermal conductivity of 1 km depth. Repeat the same estimation and calculation when shale layer does not exist (in the right).

What can be said from this calculation?

3.

Reproduce the following graph using the analytical solution.





A semi-infinite solid has initial temperature of 100 °C and the surface temperature was increased to 15°C and maintained. The analytical solution of this particular problem is list in the below and the properties of the rock is;

Thermal conductivity, k: 3.58 W/m·K, density, ρ: 1000 kg/m^3, specific heat capacity: 796 (J/kg·K).

$$\frac{T(x,t)-T_s}{T_i-T_s}=\left(\frac{2}{\sqrt{\pi}}\right)\int_0^{\eta}\exp(-u^2)du=erf(\eta)=erf\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

Due by 9am 28 Sept 2009. Please submit the hand-written homework before the lecture on Monday.