Aeroelasticity Assignment No. 1

Due Date: October 7 (Fri) 6:00 PM

1. Consider a uniform, rectangular high aspect ratio wing placed in a wind tunnel (N=1).

a) Determine the analytical expressions for the load distribution and divergence dynamic pressure of a wing. The angle of attack at the root is α_r and there is no built-in geometric twist (note that in the case of wind tunnel, N is fixed because of the support, and we can simply specify α_r). Assume two-dimensional aerodynamic strip theory (no end effects), constant geometrical and material properties, and solve the resulting differential equation exactly.

b) Considering the following numerical data:

$GJ = 6.0\ 10^4$ lb in^2	$mg = 12.5$ lb/in	$l = 60$ in
$c = 12$ in	$e = +10\%$ c	$d = +5\%$ c
$\alpha_r = 10^0$	$c_{l\alpha} = 2\pi$	

plot the elastic and rigid lift distributions as function of the dynamic pressure.

2. For the uniform, high aspect ratio wing shown on Fig. 1, derive a general expression for the influence coefficients $C_{ij}^{\theta\theta}$ and $C_{ij}^{\theta z}$ using beam theory (constitutive relations).

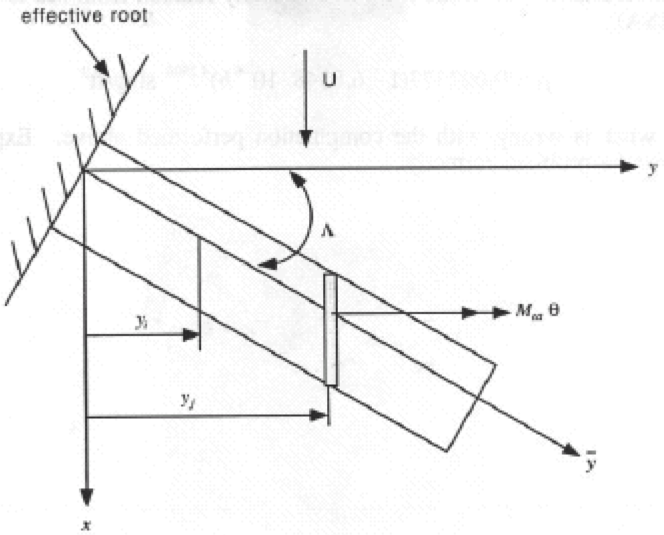


Figure 1—Swept Wing (EI, GJ = constants)

Hint: Resolve all forces and moments along $\bar{y}$ axis, and consider bending and twisting about the elastic axis. Then, resolve back to y axis.

3. For the elastic lifting surface described in the attached page:

a) Determine the torsional divergence speed as a function of altitude from sea level to 30,000 ft. Use the following matrices of flexibility influence functions, aerodynamic influence coefficients and weighting numbers for your computation.

$$\left[C^{\theta\theta}\right]=\begin{bmatrix} 424.3 & 424.3 & 424.3 & 0 \\ 424.3 & 186.6 & 186.6 & 0 \\ 424.3 & 186.6 & 78.45 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 10^{-9} \text{ rad/in lb}$$

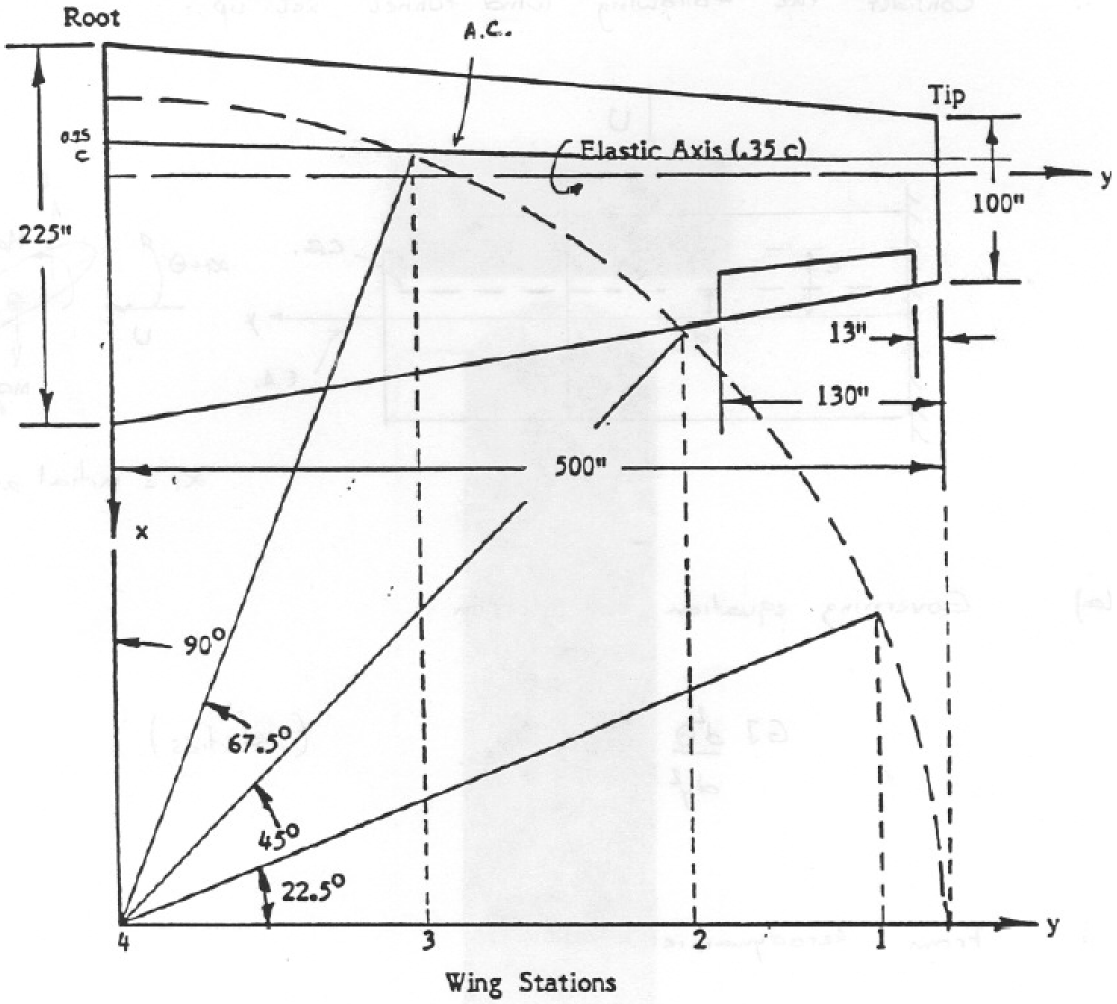
$$[A]=\begin{bmatrix} 245.25 & 90.89 & 32.31 & 17.56 \\ 49.19 & 402.38 & 125.82 & 31.69 \\ 13.38 & 96.29 & 550.29 & 130.43 \\ 13.34 & 44.82 & 241.00 & 610.50 \end{bmatrix} \text{in/rad}$$

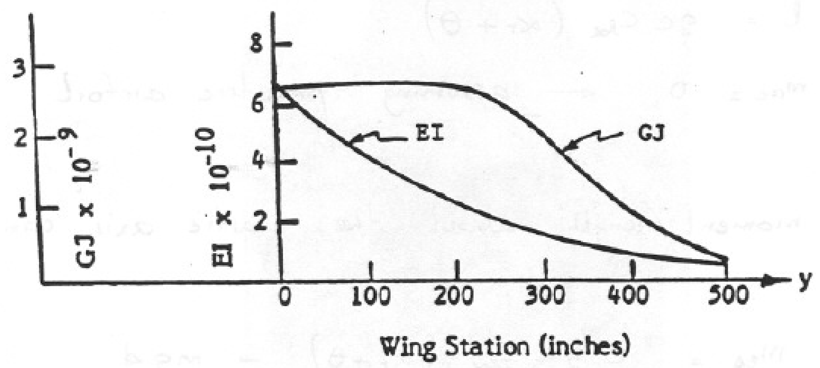
$$[W]=\begin{bmatrix} 75.14 & 0 & 0 & 0 \\ 0 & 138.84 & 0 & 0 \\ 0 & 0 & 181.40 & 0 \\ 0 & 0 & 0 & 98.17 \end{bmatrix} \text{in}$$

Note: For standard atmosphere, the altitude vs. density relation from sea level to 30,000 ft is given by (ISA):

$$\rho = 0.002377(1-6.8348\ 10^{-6}h)^{4.2586} \text{ slug/ft}^3$$

b) Describe what is wrong with the computation performed above. Explain how you could solve the problem correctly.


Root
A.C.
Tip
0.25 c
Elastic Axis (.35 c)
y
225"
100"
13"
130"
500"
x
90°
67.5°
45°
22.5°
4
3
2
1
y
Wing Stations


GJ x 10^-9
EI x 10^-10
EI
GJ
0
100
200
300
400
500
y
Wing Station (inches)