

Problem 10.10. Three-dimensional element stiffness matrix

Section 10.7.4 presents the derivation of the element stiffness matrix for a bar, leading to eq. (10.77). The presentation is limited to planar trusses; the goal of this problem is to generalize the formulation to three-dimensional (3D) problems. (1) Generalize the kinematic description of the element given in section 10.7.2. Generalize the element position vector, displacement vectors, and rotation matrix given eqs. (10.63), (10.64), and (10.66) respectively, to 3D. Select the rotation matrix as

$$\underline{\hat{R}} = \begin{bmatrix} \ell_1 - (\ell_2 + \ell_3)/\Delta & \ell_1(\ell_2 - \ell_3)/\Delta \\ \ell_2 & \ell_1/\Delta & [\ell_2(\ell_2 - \ell_3) - 1]/\Delta \\ \ell_3 & \ell_1/\Delta & [\ell_3(\ell_2 - \ell_3) + 1]/\Delta \end{bmatrix},$$

where $\Delta = \sqrt{2\ell_1^2 + (\ell_2 + \ell_3)^2}$. Prove that matrix $\underline{\hat{R}}$ is orthogonal. Express the bar's length and direction cosines in terms of the element nodal coordinates. Generalize the element coordinate transformation matrix, $\underline{\hat{T}}$, defined by eq. (10.71). (2) Generalize the expressions derived in section 10.7.3 for the element elongation and axial force, see eqs. (10.73) and (10.74), respectively. (3) Generalize the expressions for the element strain energy and stiffness matrix given in section 10.7.4. Give the expression for the stiffness matrix in the local and global coordinate systems, see eqs. (10.75) and (10.77), respectively. (4) The stiffness matrix expressed in the global coordinate system is of size 6×6 . Of the six eigenvalues of this matrix, how many are zero? Discuss the nature of the corresponding eigenvectors.

Problem 10.11. Global stiffness matrix assembly process

Figure 10.24 gives a pictorial representation of the assembly process for the truss and node numbering sequence shown in fig. 10.22. Give the corresponding representation of the assembly process for the same truss using the node numbering shown in fig. 10.25.

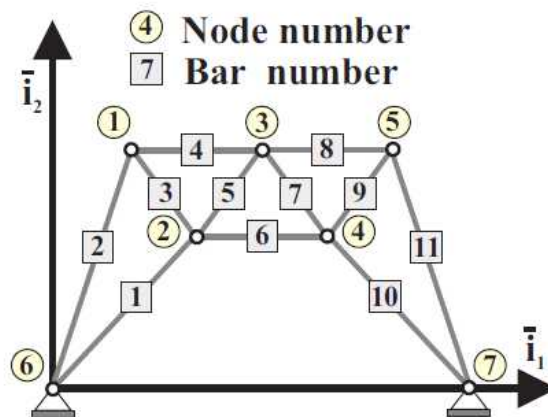


Fig. 10.25. Eleven-bar truss.