Problem 11.2. Cantilever with tip load: polynomial solution
Consider a cantilever beam of length L and bending stiffness H_{33}^c subjected to a transverse concentrated load, P, acting at the beam's tip. *(1)* Construct a one-term monomial solution, $\bar{u}_2(\eta) = \eta^2 q_1$, and compare the computed tip displacement to the exact value of $PL^3/(3H_{33}^c)$. *(2)* Construct a two-term monomial solution, $\bar{u}_2(\eta) = \eta^2 q_1 + \eta^3 q_2$, and compare this with the exact value. *(3)* Compute the bending moment M_3 and the shear V_2 at the root using the two-term solution and compare these values to the exact values, which can readily be determined from statics.

Problem 11.4. Cantilever beam with elliptical pressure load
Consider a cantilever beam of length L and bending stiffness H_{33}^c subjected to a transverse distributed load $p_2(\eta) = p_0\sqrt{1-\eta^2}$ that simulates the aerodynamic load acting on an aircraft wing of semi-span L. *(1)* Develop a one-term approximate solution and compare the tip deflection with the exact result determined using the unit load method. *(2)* Repeat the development for a two-term solution. Hint: follow the approach and shape functions used in example 11.1

Problem 11.6. Rotating helicopter blade with tip mass
A helicopter blade of length L and with a tip mass M_0 is rotating at an angular velocity Ω about axis $\bar{\imath}_2$, see fig 11.21. The blade is homogeneous and its cross-section linearly tapers from an area $\mathcal{A}_0$ at the root to $\mathcal{A}_1$ at the tip so that $\mathcal{A}(x_1) = \mathcal{A}_0 + (\mathcal{A}_1 - \mathcal{A}_0)x_1/L$. Select $\mathcal{A}_0 = 2\mathcal{A}_1$. The tip mass $M_0 = \zeta\rho\mathcal{A}_0 L$, where $\zeta = 0.2$ and ρ is the material mass density. *(1)* Solve the governing differential equations of this problem to find the axial displacement $\bar{u}(x_1)$ and the axial load $N_1(x_1)$. *(2)* Find an approximate solution for the axial displacement $\bar{u}_1(x_1)$ using a weak formulation. Select the following forms for the displacement field, $\bar{u}_1(x_1) = q_1 x_1 + q_2 x_1^2$, and weighting function, $w(x_1) = w_1 x_1 + w_2 x_1^2$. *(3)* Determine the axial force $N_1(x_1)$. *(4)* On the same graph, plot the non-dimensional displacement fields for the exact and approximate solutions. *(5)* On the same graph, plot the non-dimensional axial force for the exact and approximate solutions. *(6)* How would you improve the approximate solution? **Hint**: A mass M rotating about axis $\bar{\imath}_2$ at an angular velocity Ω is subjected to a centrifugal force $F_c = M\Omega^2 r$, where r is the distance between the mass and the axis of rotation. Hence, the helicopter blade is subjected to an axial load per unit span $p_1(x_1) = \rho\mathcal{A}(x_1)\Omega^2 x_1$, where ρ is the material density. In a similar way, the tip mass M_0 creates a concentrated tip force $M_0\Omega^2 L$.

Problem 11.9. Uniformly loaded simply supported beam

Consider a simply supported, uniform beam of length L subjected to a uniform transverse loading $p_2(x_1) = p_0$, as depicted previously in fig. 11.24. *(1)* Solve the governing differential equations of this problem to find the transverse displacement $\bar{u}_2(x_1)$, the bending moment $M_3(x_1)$, and the shear force $V_2(x_1)$. *(2)* Find an approximate solution of the problem using a weak formulation. Select the following forms for the displacement field $\bar{u}_2(x_1) = \sum_{i=1}^{N} q_i \sin(2i-1)\pi x_1/L$ and test function $w(x_1) = \sum_{i=1}^{N} w_i \sin(2i-1)\pi x_1/L$. *(3)* Plot the exact and approximate transverse displacement fields $\bar{u}_2(x_1)$ on the same plot. For the approximate solutions use $N = 1, 2, 3, 4$, and 5. *(4)* Plot the exact and approximate bending moments $M_3(x_1)$ on the same plot. *(5)* Plot the exact and approximate shear forces $V_2(x_1)$ on the same plot.

Problem 11.10. Simply supported beam with concentrated load

Consider a simply supported beam with a concentrated load, P, applied at a point $x_1 = \alpha L$ from the left support as illustrated in fig. 5.23 on page 197. This configuration is solved using the classical differential equation approach in example 5.5, and the transverse displacement is found to be given by eq. (5.51). The solution presents a discontinuity in the transverse shear force, and solutions are developed separately for the portions of the beam to the left and right of the concentrated load. Using the weak statement, it is possible to develop a single expression that approximates the deflection over the entire span of the beam, because the continuity requirements associated with this approach are lower than those required for the differential equation approach. *(1)* Find an approximate solution of the problem using a weak formulation. Select the following forms for the displacement field $\bar{u}_2(x_1) = \sum_{i=1}^{N} q_i \sin i\pi x_1/L$ and test function $w(x_1) = \sum_{i=1}^{N} w_i \sin i\pi x_1/L$. *(2)* On one graph, plot the exact and approximate transverse displacement fields, $H_{33}^c \bar{u}_2(x_1)/(PL^3)$. For the approximate solutions, use $N = 1, 2, 3, 4$, and 5. *(3)* On one graph, plot the exact and approximate bending moment distributions, $M_3(x_1)/(PL)$. *(4)* On one graph, plot the exact and approximate shear force diagrams, $V_2(x_1)/P$.