Problem 11.11. Cantilever with nonuniform bending stiffness

Consider the cantilevered beam subjected to a tip load P as shown in fig. 11.35. The bending stiffness of the beam's left half is $3H_0$, while that of the right half is H_0, as shown in the figure. Develop an approximate solution for the transverse deflection of the entire beam using the principle of minimum total potential energy with a two-term polynomial. Compare your solution at the tip with the exact solution computed using the unit load method.

Problem 11.12. Simply-supported beam with nonuniform bending stiffness

Consider the cantilever beam shown in fig. 11.35 but now assume that both ends are simply supported instead. The bending stiffness of the beam's right half is H_0 while that of the left half is βH_0 where $\beta = 3$. Develop an approximate solution for the transverse deflection of the entire beam using the principle of minimum total potential energy with a two-term trigonometric approximate solution. Compare your solution at the mid-span with the exact solution computed using the unit load method.

Problem 11.14. Simply-supported beam with two mid-span springs

Consider a simply supported, uniform beam of length L with two end point torsional springs of stiffness k_1 and a mid-span spring of stiffness k_2. The beam, shown in fig. 11.37, is subjected to a uniform transverse loading $p_2(x_1) = p_0$. *(1)* Solve the governing differential equations of this problem to find the transverse displacement $\bar{u}_2(x_1)$, the bending moment $M_3(x_1)$, and the shear force $V_2(x_1)$. *(2)* Find an approximate solution of the problem using the principle of minimum total potential energy. Select the following form for the displacement field: $\bar{u}_2(x_1) = q_1 \sin \pi x_1/L + q_3 \sin \pi 3x_1/L$. *(3)* On the same graph, plot the exact and approximate transverse displacement fields, $H_{33}^c \bar{u}_2/(p_0 L^4)$. *(4)* On the same graph, plot the exact and approximate bending moment distributions, $M_3/(p_0 L^2)$. *(5)* On the same graph, plot the exact and approximate shear force distributions, $V_2/(p_0 L)$. *(6)* Explain why the approximation is so poor. Hint: look at the bending moment plots. It will be convenient to work with non-dimensional spring stiffnesses $\bar{k}_1 = k_1 L/H_{33}^c$ and $\bar{k}_2 = k_2 L^3/H_{33}^c$. For your plots, select $\bar{k}_1 = 10.0$ and $\bar{k}_2 = 100.0$.

Problem 11.16. Simply supported beam with variable bending stiffness

A simply supported beam of span L is subjected to forces of magnitude P located at stations $x_1 = \alpha L$ and $(1-\alpha)L$, as depicted in fig. 11.39. The beam has a bending stiffness H_0 and is reinforced in its central portion where its bending stiffness is H_1. *(1)* Find the exact solution of the problem from the solution of the governing differential equation and associated boundary conditions. *(2)* Use the principle of minimum total potential energy to find approximate solutions for this problem using the following shape functions: $h_i(x_1) = \sin(2i-1)\pi x_1/L$ using the first 1, 2 and 3 terms. On a single graph, plot the exact solution and the 3 approximate solutions. Also, construct a single plot of the error in maximum displacement for the 3 approximate solutions. Use $H_1/H_0 = 2$ and $\alpha = 0.3$. *(3)* Find the bending moment distribution for the problem. On a single graph, plot the exact solution and the 3 approximate solutions using. Also, construct a single plot of the error in maximum bending moment for the approximate solutions. *(4)* Based on a simple free body diagram, show that for the exact solution the shear force presents a discontinuity at the point of application of the transverse loads P. What happens in your approximate solution? Comment and explain your results. On a single graph, plot the exact solution and the approximate shear force distribution for the approximate solutions.