Problem 12.1. Cantilevered beam with elastic foundation

A cantilevered beam of length L is subjected to a tip load P_2, a tip bending moment Q_3, a transverse distributed load $p_2(x_1)$, and a distributed bending moment $q_3(x_1)$, as shown in fig. 12.12. The cantilevered beam is supported by an elastic foundation of stiffness k, not shown on the figure, for clarity. The total potential energy of the system is

$$\Pi = \int_0^L \left[\frac{1}{2} H_{33} \left(\frac{\mathrm{d}^2 u_2}{\mathrm{d}x_1^2} \right)^2 + \frac{1}{2} k \, u_2^2 \right] \mathrm{d}x_1 - \int_0^L \left(p_2 \, u_2 + q_3 \frac{\mathrm{d}u_2}{\mathrm{d}x_1} \right) \mathrm{d}x_1$$
$$- P_2 \, u_2(L) - Q_3 \left. \frac{\mathrm{d}u_2}{\mathrm{d}x_1} \right|_L .$$

(1) Find the governing differential equations and boundary conditions for this problem using the principle of minimum total potential energy. *(2)* Derive the same equations and boundary conditions based on simple free body diagrams for a differential element of the beam.

** Do not solve Problem 12.1.(2)

Problem 12.3. Cantilevered beam with various loading

The uniform cantilevered beam of span L depicted in fig. 12.13 has a bending stiffness H_{33} and is supported by an elastic foundation of stiffness k over its first half. A concentrated spring of stiffness k_1 supports the beam at its free end. A mid-span concentrated load P is applied together with a uniform distributed load p_0 that acts over the second half of the beam span. Write the principle of minimum total potential energy for this system.

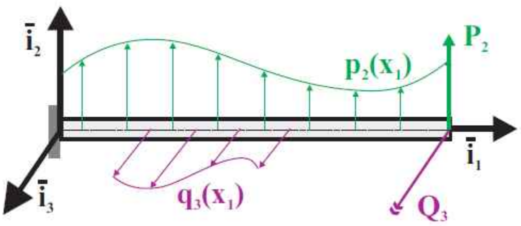

Fig. 12.12. Cantilevered beam with concentrated an distributed moments.

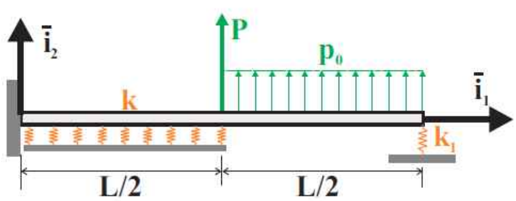

Fig. 12.13. Simply supported beam with partial elastic foundation.

Problem 12.5. Cantilevered beam with tip spring

The uniform cantilevered beam of span L shown in fig. 12.14 features a tip spring of stiffness k and a tip concentrated load P. Write the principle of minimum total potential energy for the system. From this principle, derive the governing differential equations of the problem and the associated boundary conditions. Explain the physical meaning of the boundary conditions at $x_1 = L$ using a free body diagram.

Problem 12.6. Simply supported beam with end torsional springs

Consider a simply supported, uniform beam of length L with two end point torsional springs of stiffness k_1 and a mid-span spring of stiffness k_2. The beam, shown in fig. 12.15, is subjected to a uniform transverse loading $p_2(x_1) = p_0$. Write the principle of minimum total potential energy for the system. From this principle, derive the governing differential equations of the problem and the associated boundary conditions. Explain the physical meaning of the boundary conditions at $x_1 = L/2$ using a free body diagram.

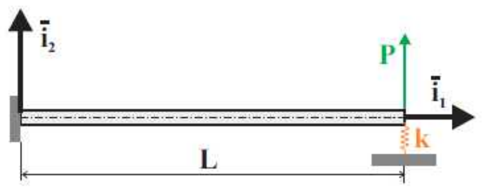

Fig. 12.14. Cantilevered beam with tip concentrated load and elastic spring.

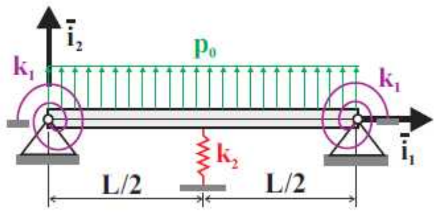

Fig. 12.15. Simply supported beam with mid-span and end point springs.