

# Self-Attention Graph-Pooling (SAGP)

Yeongtak Oh

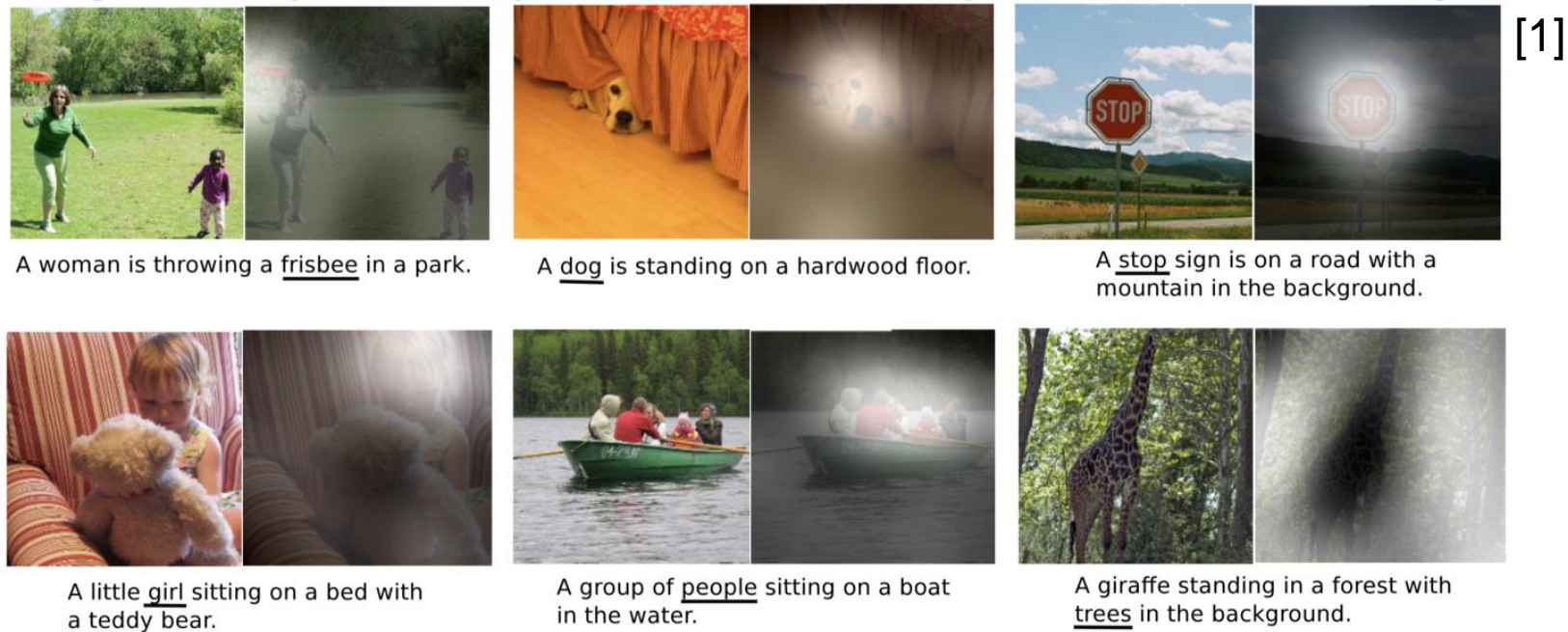
Seoul National University

Lee, Junhyun, Inyeop Lee, and Jaewoo Kang. "Self-attention graph pooling.", ICML 2019, Citation: 42

# Self-Attention Graph-Pooling

## ■ Introduction

- 1) Attention : used for feature selection based on multi-modal relationships of data<sup>[1]</sup>
- 2) Self-attention : allows input features to be the criteria for the attention itself<sup>[2]</sup>



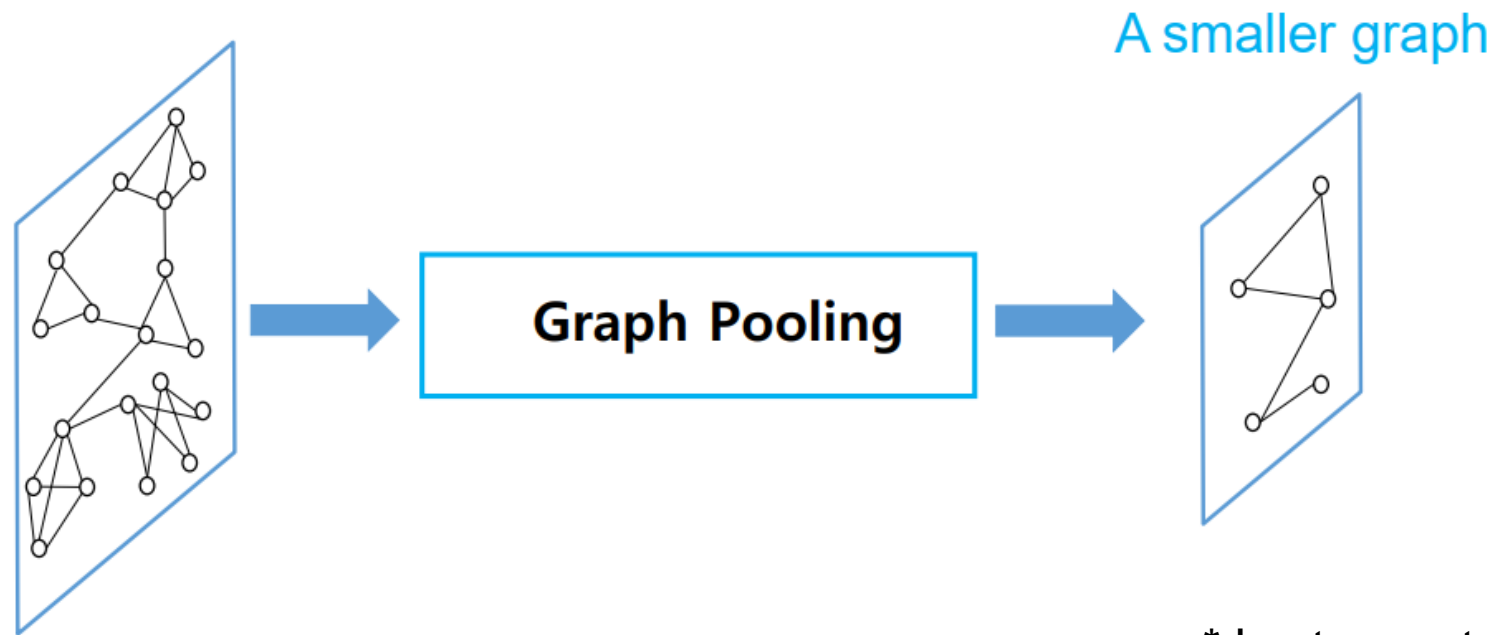
[1] Xu, Kelvin, et al. "Show, attend and tell: Neural image caption generation with visual attention." *ICML*. 2015.

[2] Vaswani, Ashish, et al. "Attention is all you need." *NIPS*. 2017.

# Self-Attention Graph-Pooling

## ■ Introduction

### 3) Graph Pooling



\* Lecture note 13, p 15

- Intuition: **Down-sample** by selecting the most important nodes
- # of nodes: **decrease**, dimension of graphs: **consistent**

# Self-Attention Graph-Pooling

## ■ Proposed Method

### 1) Graph U-nets: gPool

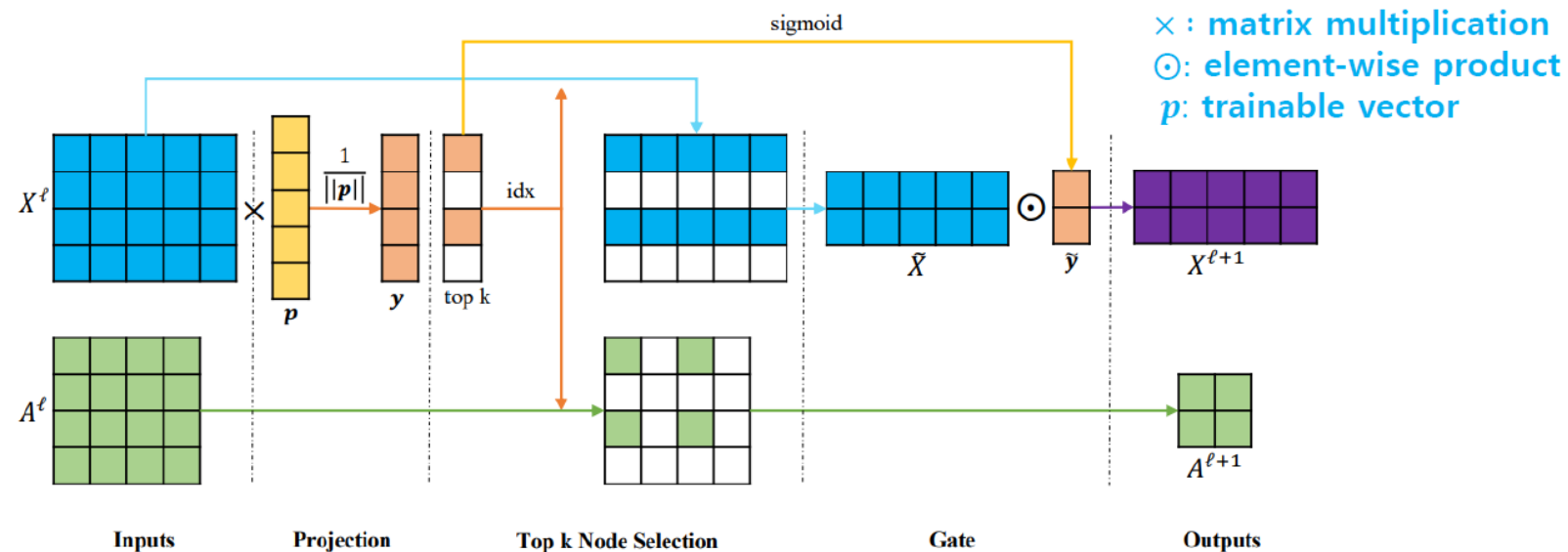
$$y = \text{softmax}(X_p)$$

$$i = y_i > \alpha$$

$$X' = (X \odot y)_i$$

$$A' = A_{i,i}$$

#### Top-k graph pooling



[3] Gao, Hongyang, and Shuiwang Ji. "Graph u-nets." arXiv preprint arXiv:1905.05178 (2019).

# Self-Attention Graph-Pooling

## ■ Proposed Method

### 1) Self-Attention Graph Pooling

$$y = \text{softmax}(GNN(X, A))$$

$$i = y_i > \alpha$$

$$X' = (X \odot y)_i$$

$$A' = A_{i,i}$$

Self-attention graph pooling

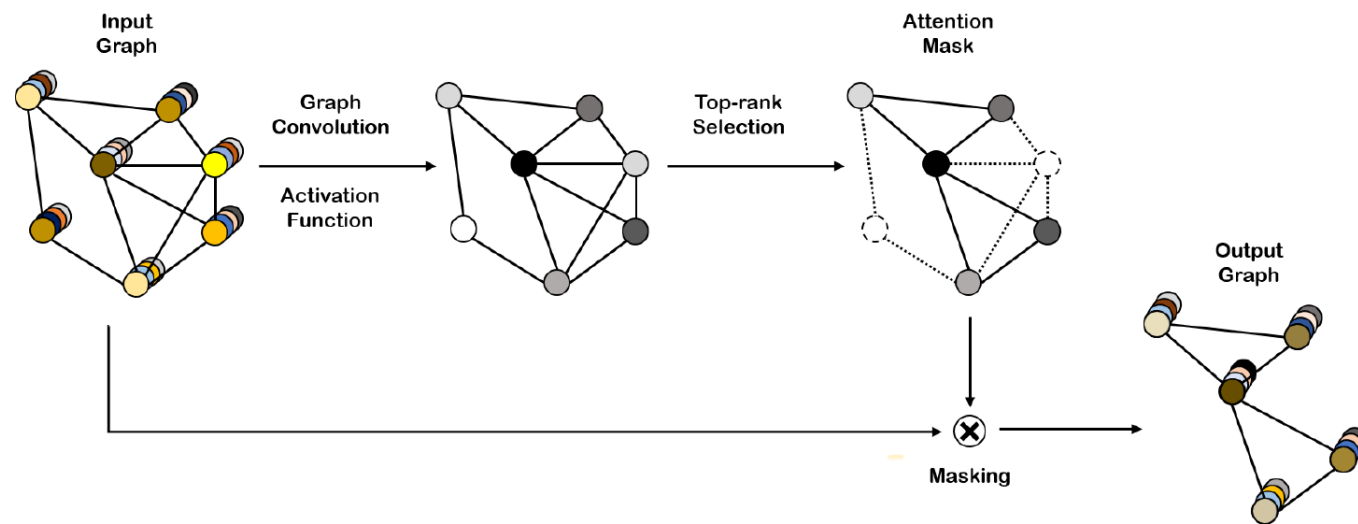


Figure 1. An illustration of the SAGPool layer.

# Self-Attention Graph-Pooling

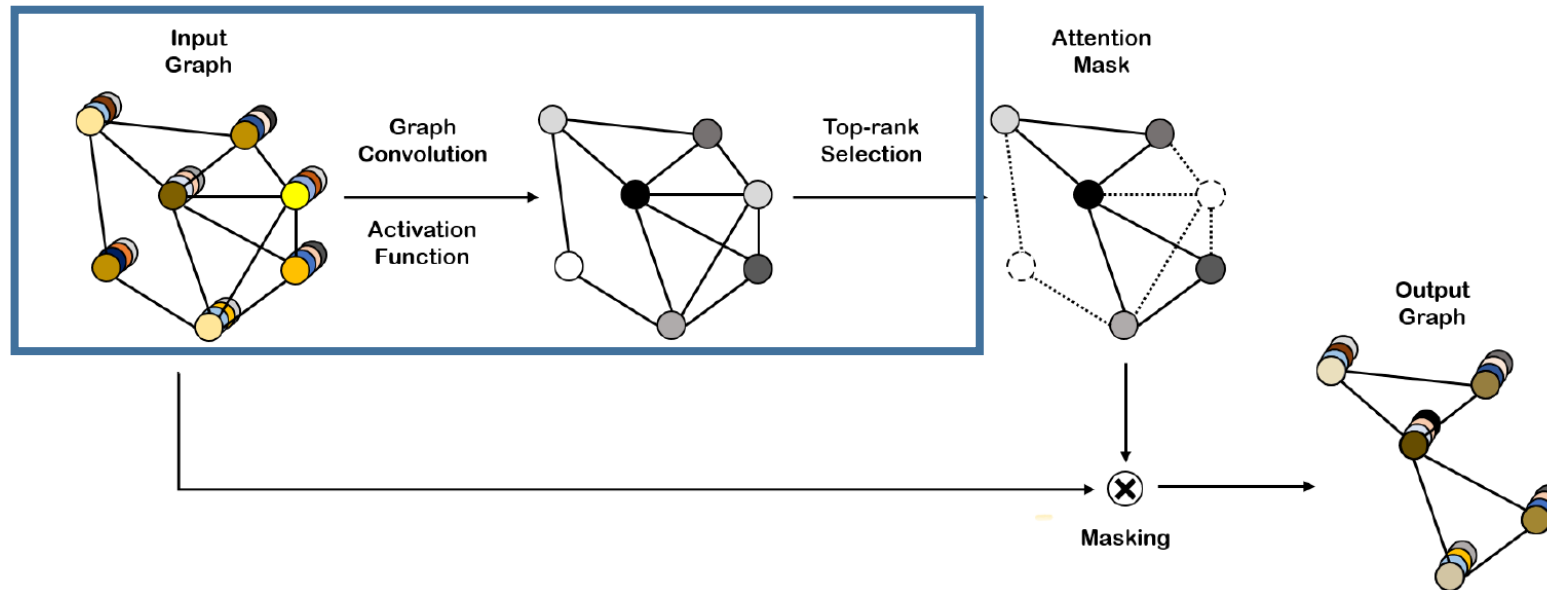
## ■ Proposed Method

### 1) Self-Attention Score: Utilizing the graph convolution

- $Z = \sigma \left( \tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} X \Theta_{att} \right), X \in \mathbb{R}^{N \times F}, \Theta_{att} \in \mathbb{R}^{F \times 1}$
- $idx = top - rank(Z, [kN]), Z_{mask} = Z_{idx}$

\* Remind of Graph Convolution

$$h^{(l+1)} = \sigma(\tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} h^{(l)} \Theta), \Theta \in \mathbb{R}^{F \times F'}$$



# Self-Attention Graph-Pooling

## ■ Proposed Method

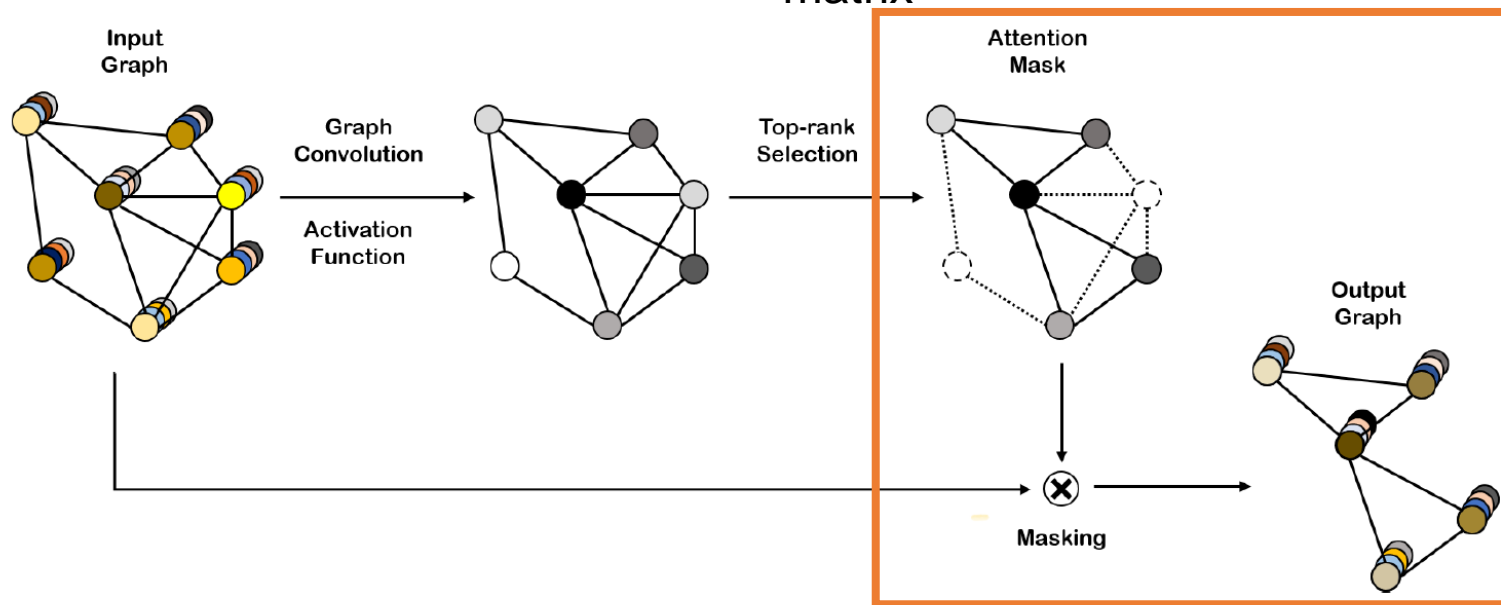
### 2) Graph Pooling

- $X' = X_{idx}, X_{out} = X' \odot Z_{mask}, A_{out} = A_{idx,idx}$

Node-wise  
feature matrix

Element-wise  
Product

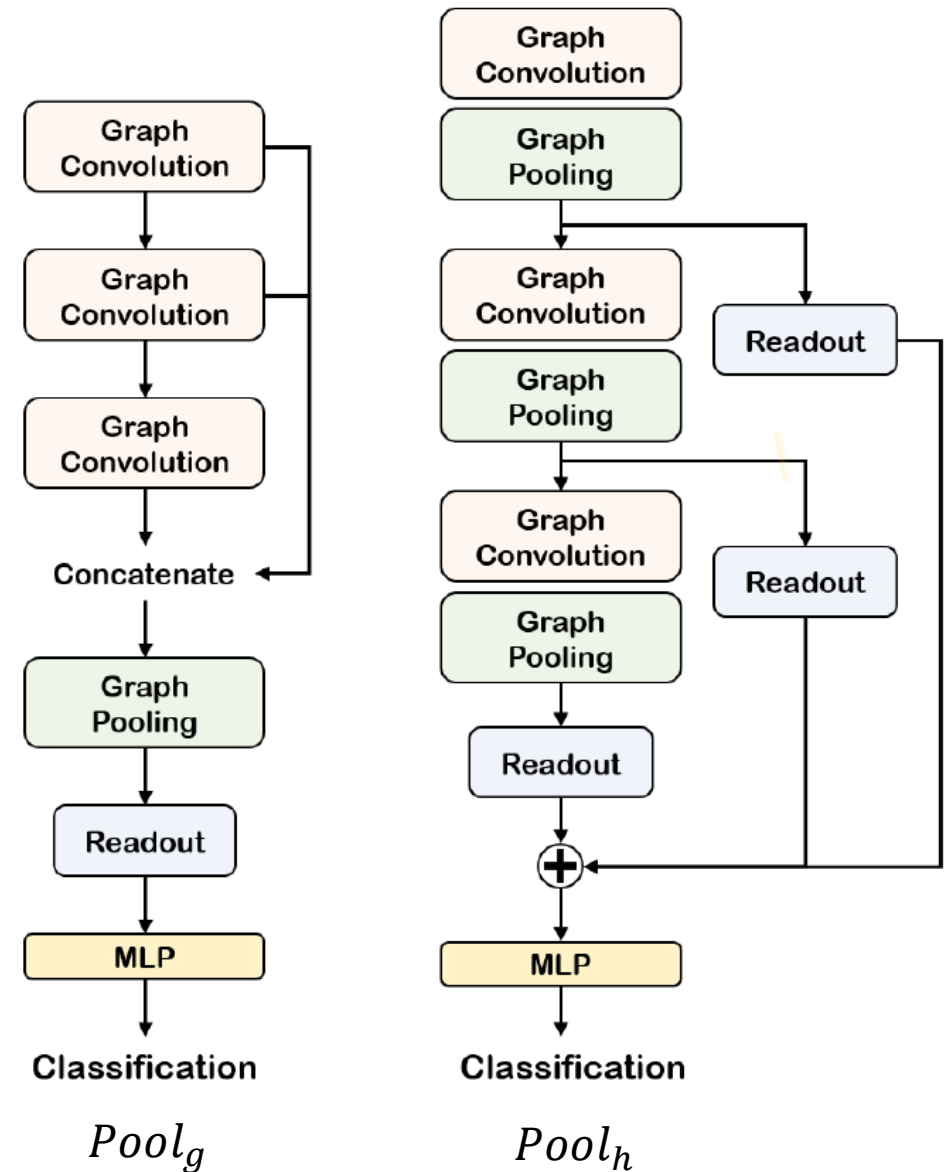
Indexed-Adjacency  
matrix



# Self-Attention Graph-Pooling

- Proposed method
- Global Pooling as,  $Pool_g$
- Hierarchical Pooling as,  $Pool_h$
- Readout Layer: Aggregates node features

$$s = \frac{1}{N} \sum_{i=1}^N x_i || \max_{i=1}^N x_i$$





# Self-Attention Graph-Pooling

## ■ Results

- 1) SOTA of Both on  $Pool_g$  and  $Pool_h$  architecture
- 2)  $Pool_g$  for smaller graph,  $Pool_h$  for a large number of nodes

Table 3. Average accuracy and standard deviation of the 20 random seeds. The subscript  $g$  (e.g.  $POOL_g$ ) denotes the global pooling architecture and the subscript  $h$  (e.g.  $POOL_h$ ) denotes the hierarchical pooling architecture.

Models	D&D	PROTEINS	NCI1	NCI109	FRANKENSTEIN
Set2Set <sub>g</sub>	71.27 ± 0.84	66.06 ± 1.66	68.55 ± 1.92	69.78 ± 1.16	61.92 ± 0.73
SortPool <sub>g</sub>	72.53 ± 1.19	66.72 ± 3.56	73.82 ± 0.96	74.02 ± 1.18	60.61 ± 0.77
SAGPool <sub>g</sub> (Ours)	<b>76.19 ± 0.94</b>	<b>70.04 ± 1.47</b>	<b>74.18 ± 1.20</b>	<b>74.06 ± 0.78</b>	<b>62.57 ± 0.60</b>
DiffPool <sub>h</sub>	66.95 ± 2.41	68.20 ± 2.02	62.32 ± 1.90	61.98 ± 1.98	60.60 ± 1.62
gPool <sub>h</sub>	75.01 ± 0.86	71.10 ± 0.90	67.02 ± 2.25	66.12 ± 1.60	61.46 ± 0.84
SAGPool <sub>h</sub> (Ours)	<b>76.45 ± 0.97</b>	<b>71.86 ± 0.97</b>	<b>67.45 ± 1.11</b>	<b>67.86 ± 1.41</b>	<b>61.73 ± 0.76</b>

← Larger graph

# Self-Attention Graph-Pooling

## ■ Results

- 1) SOTA of Both on  $Pool_g$  and  $Pool_h$  architecture
- 2)  $Pool_g$  for smaller graph,  $Pool_h$  for a large number of nodes

Table 1. Statistics of data sets.

Data set	Number of Graphs	Number of Classes	Avg. # of Nodes per Graph	Avg. # of Edges per Graph
D&D	1178	2	284.32	715.66
PROTEINS	1113	2	39.06	72.82
NCI1	4110	2	29.87	32.30
NCI109	4127	2	29.68	32.13
FRANKENSTEIN	4337	2	16.90	17.88

Hyperparameter	Range
Learning rate	1e-2, 5e-2, 1e-3, 5e-3, 1e-4, 5e-4
Hidden size	16, 32, 64, 128
Weight decay (L2 regularization)	1e-2, 1e-3, 1e-4, 1e-5
Pooling ratio	1/2, 1/4

# Self-Attention Graph-Pooling

## ■ Results

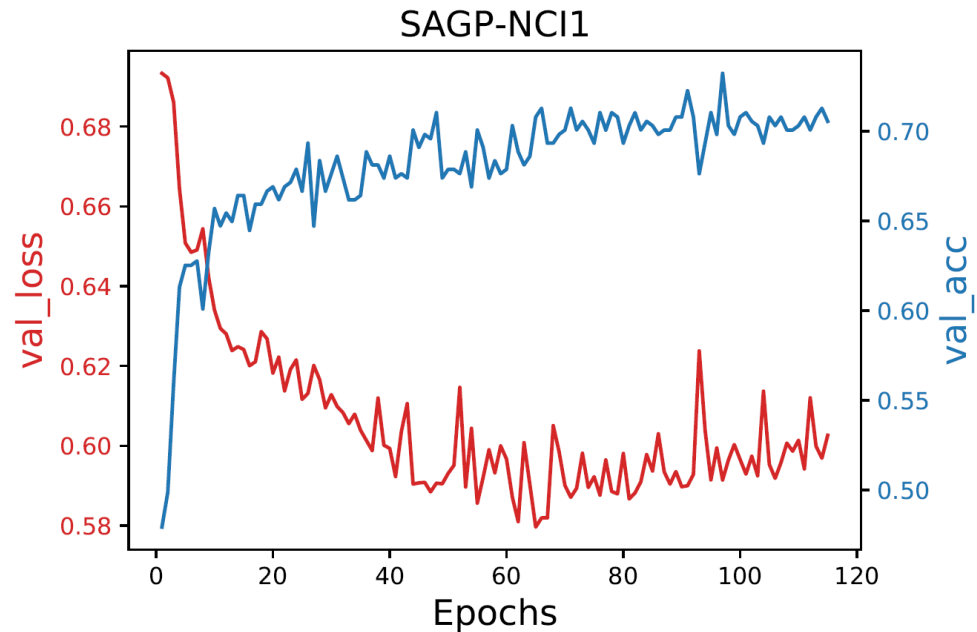
- 1) SOTA of Both on  $Pool_g$  and  $Pool_h$  architecture
- 2)  $Pool_g$  for smaller graph,  $Pool_h$  for a large number of nodes

Graph Convolution	D&D	PROTEINS
SAGPool <sub>h</sub>	76.45 ± 0.97	71.86 ± 0.97
SAGPool <sub>h</sub> ,Cheb	75.82 ± 0.79	71.98 ± 0.93
SAGPool <sub>h</sub> ,SAGE	76.28 ± 1.06	71.93 ± 0.82
SAGPool <sub>h</sub> ,GAT	75.49 ± 0.93	71.98 ± 1.01
SAGPool <sub>h</sub> ,augmentation	77.07 ± 0.82	71.82 ± 0.81
SAGPool <sub>h</sub> ,serial,2layers	76.68 ± 0.96	72.17 ± 0.87
SAGPool <sub>h</sub> ,parallel,M=2	75.79 ± 0.96	72.05 ± 0.43
SAGPool <sub>h</sub> ,parallel,M=4	76.77 ± 0.61	71.66 ± 0.98

# Self-Attention Graph-Pooling

## ■ Reproducing Results

- AIDS, DD, PROTEINS, NCI1 dataset에 대해 reproducing experiment를 수행
- Double-axis로 val\_loss 및 val\_acc 값을 plot



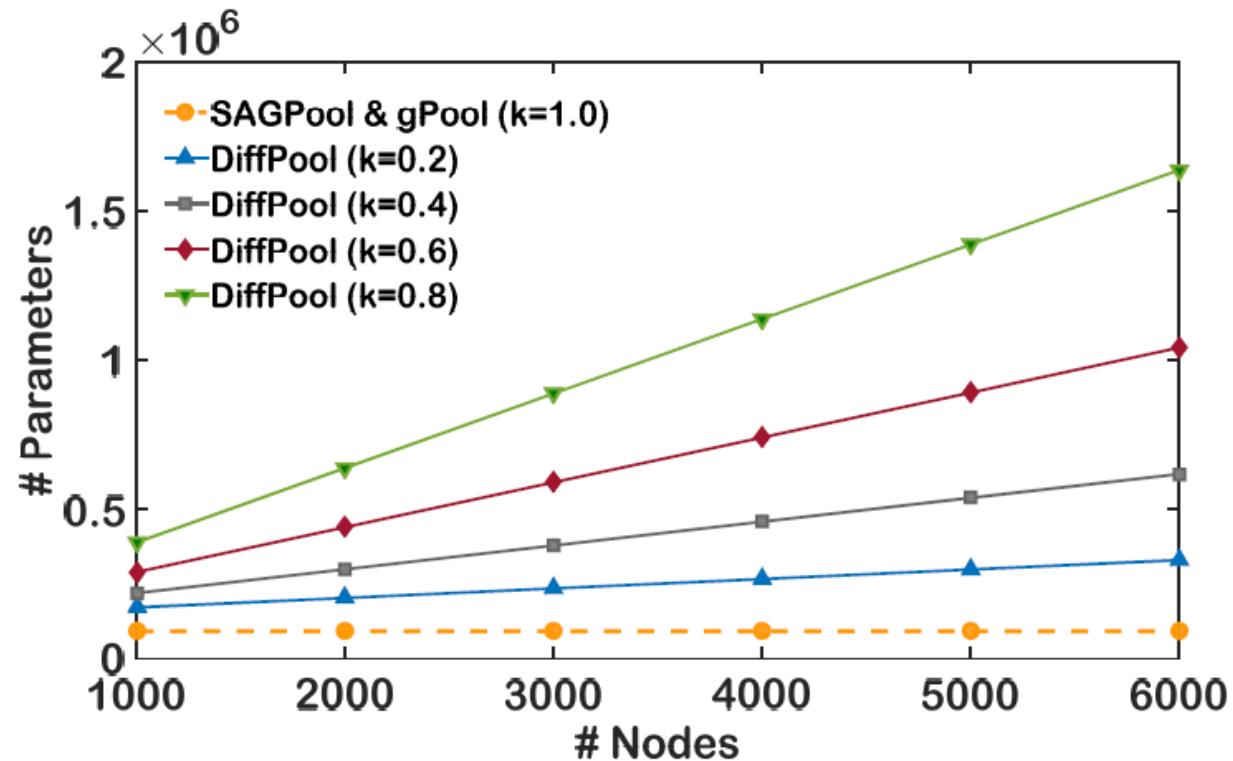
Datasets: <https://chrsmrrs.github.io/datasets/docs/datasets/>

Reproducing Code: [https://github.com/oyt9306/SAGP\\_reproducing](https://github.com/oyt9306/SAGP_reproducing)

# Self-Attention Graph-Pooling

## ■ Results

- 3) Same as  $O(|V| + |E|)$  of gPool, but  $O(|V|^2)$  of DiffPool
- 4) Consistent number of parameters regardless of the input



# Conclusions and Remarks

- Conclusion:

- 1) Applying the concept of self-attention into a graph pooling
- 2) Showed a reasonable complexity, and end-to-end representation learning
- 3) Possible to expand with many variants(e.g. with SAGE, GAT)

- Future works:

Learnable pooling ratio, optimal cluster size, multiple attention mask

# Appendix

- Additional References about Graph Pooling:

- 1) Diehl, Frederik. "Edge contraction pooling for graph neural networks." arXiv preprint arXiv:1905.10990 (2019).
- 2) Ranjan, Ekagra, Soumya Sanyal, and Partha Pratim Talukdar. "ASAP: Adaptive Structure Aware Pooling for Learning Hierarchical Graph Representations." arXiv preprint arXiv:1911.07979 (2019).

# Thank you for Listening