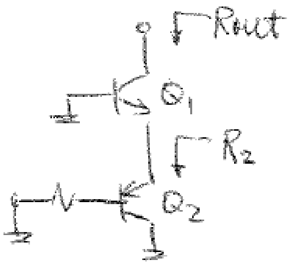
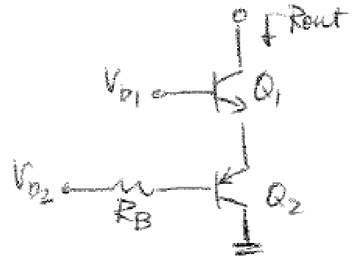
1.

Chapter9. 8

(a)





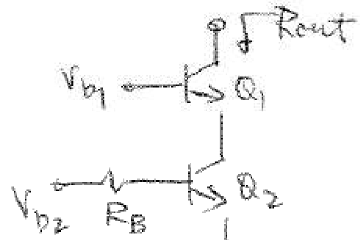
(Ac-small signal)

Looking into emitter of Q2,

$$R_2 = \frac{1}{\left(\frac{\beta+1}{R_B + r_{\pi_2}} + \frac{1}{r_{o2}}\right)}$$

$$\Rightarrow R_{out} = \left[1 + g_{m_1}(R_2 \parallel r_{\pi_1})\right] r_{o_1} + (R_2 \parallel r_{\pi_1})$$

(b) R_B does not affect Q_2 in small-signal R_{out}.



$$\therefore R_{out} = \left[1 + g_{m_1}(r_{o2} \parallel r_{\pi_1})\right] r_{o_1} + (r_{o2} \parallel r_{\pi_1})$$

This is a cascode stage.

Chapter9. 19

$$A_v = -g_{m1} r_{o1} g_{m1} (r_{o1} \parallel r_{\pi 2})$$

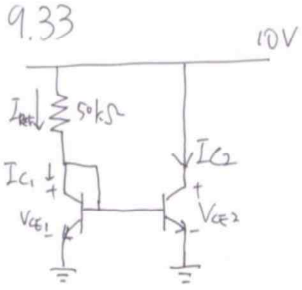
$$= -\frac{I_{C1}}{V_T} \cdot \frac{V_{A1}}{I_{C1}} \cdot \frac{I_{C1}}{V_T} \cdot \frac{1}{\frac{I_{C1}}{V_{A1}} + \frac{I_{C2}}{\beta V_T}}$$

Since $I_{C1} \approx I_{C2}$,

$$A_v \approx -\frac{V_{A1}/V_T^2}{\frac{1}{V_{A1}} + \frac{1}{\beta V_T}} = -\frac{\beta V_A^2}{V_T (V_A + \beta V_T)}$$

Chapter9. 33

9.33



$\beta = 75,\quad I_s = 2\times10^{-16}A,\quad V_T = 26mV,\quad V_A = 50mV$

$$I_{C1} = I_s \exp\left(\frac{V_{CE1}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$

$$\Rightarrow V_T \ln\left[\frac{I_c}{I_s}\,\frac{V_A}{V_A+V_{CE1}}\right] = V_{CE} \quad \cdots ①$$

$$I_{REF} = I_c\left(1+\frac{2}{\beta}\right)$$ (Assume two bjts are identical)

$$V_{CE1} = 10 - 50\times10^3 I_{REF} = 10 - 50\times10^3 \times I_c \times \frac{77}{75} \quad \cdots ②$$

From ① and ②, $V_{CE1} = 715.4mV$.

$$I_{C2} = I_s \exp\frac{0.7154}{0.026}\times\left(1+\frac{10}{50}\right)$$

$$= \underline{0.214mA}$$

Chapter9. 40

9.40

$$I_{REF} = \frac{V_{DD} - V_{GS1}}{R} = 2mA.$$

$$2mA = \frac{1}{2} k_n \frac{W}{L} (V_{GS1} - V_{th})^2$$

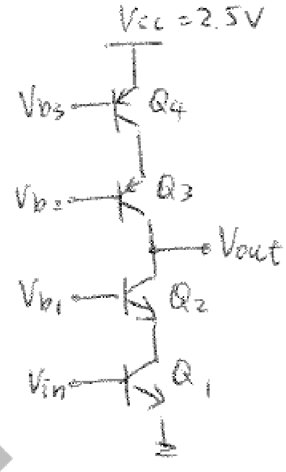
$$= \frac{1}{2} \times 20 \mu A \times \left(\frac{W}{L}\right)_1 \times (1 - 0.4)^2.$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 556$$

Given power budget = 2mW

$V_{BC_1} = V_{CB_4} = 200\ mV$,

calculate voltage gain.



$$\alpha_p = \frac{50}{50+1} \approx 0.98$$

$$\alpha_n = \frac{100}{100+1} \approx 0.99$$

$\therefore$ we assume $I_{c,p} \approx I_{e,p}$ & $I_{c,n} \approx I_{e,n}$

This implies that $I_{BIAS} = \frac{Power}{V_{CC}} = \frac{2mW}{2.5V} \approx 0.8mA$.

$$\Rightarrow V_{BE_1} = V_{in} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V) \cdot \ln\left(\frac{0.8mA}{6 \cdot 10^{-16} A}\right) \approx 0.726V$$

$$V_{C,1} = V_{BE_1} - V_{BC_1} = 0.726V - 0.2V = 0.526V$$

$$\therefore V_{b_1} = V_{C_1} + V_{BE_2} = (0.526V) + (0.026V) \ln\left(\frac{0.8mA}{6 \cdot 10^{-6} A}\right)$$

$$\approx 1.252V$$

$$\Rightarrow V_{EB_4} = V_{CC} - V_{b3} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,4}}\right) = 0.026V \cdot \ln\left(\frac{0.8mA}{6 \cdot 10^{-16} A}\right)$$

$$\approx 0.726V$$

$$V_{b3} = V_{CC} - 0.726V = 1.774\ V$$

$$V_{C4} = V_{D3} + V_{CB4} = 1.774V + 0.2V = 1.974\ V$$

$$\therefore V_{D2} = V_{C4} - V_{EB3} = (1.974V) - (0.026)\ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right)$$

$$\approx 1.248\ V$$

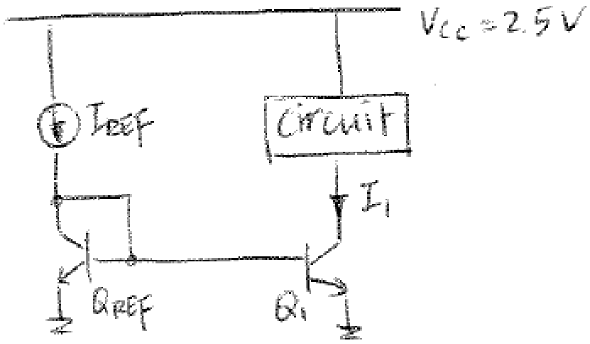
$$A_V = -g_{m1}\left\{\left[g_{m2}r_{o2}(r_{o1} \| r_{\pi 2})\right] \| \left[g_{m3}r_{o3}(r_{o4} \| r_{\pi 3})\right]\right\}$$

After simplifying, A_V is independent of I_{BIAS}:

$$A_V \approx \frac{V_{AN} \cdot V_{AP}}{V_T^2\left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_P V_T}\right)}$$

$$= \frac{5 \cdot 5}{(0.026V)^2\left(\frac{5}{5} + \frac{5}{100 \cdot 0.026} + \frac{5}{5} + \frac{5}{50 \cdot 0.026}\right)}$$

$$\approx 4760$$



$I_1 = 0.5mA$
power = 2mW

$$\text{power} = V_{CC}(I_{REF} + I_1)$$

$$\Rightarrow I_{REF} = \frac{\text{Power}}{V_{CC}} - I_1 = \frac{2mW}{2.5V} - 0.5mA = 0.3mA$$

Therefore, if Q_{REF} has area A_E, then Q_1 has area $\frac{5}{3}A_E$ for the currents specified.

i.e. $\frac{A_{REF}}{A_1} = \frac{3}{5}$

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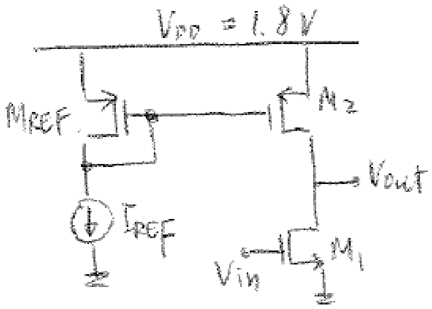

$A_V = -20$

power = 2mW

$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\quad \lambda_n = 0.1V^{-1}$

$\lambda_p = 0.2V^{-1}$

$$R_{out} = r_{o2} \parallel r_{o1} = \frac{1}{\lambda_n I_{D_1} + \lambda_p I_{D_1}}$$

$$\Rightarrow A_V = -G_m R_{out} = -\frac{g_{m_1}}{\lambda_n I_{D_1} + \lambda_p I_{D_1}} = -\frac{2I_{D_1}/(V_{GS_1}-V_{TH})}{I_{D_1}(\lambda_n + \lambda_p)}$$

$$\Rightarrow -20 = -\frac{2}{(V_{GS_1}-V_{TH})(\lambda_n+\lambda_p)}$$

$$\Rightarrow V_{GS_1} = \frac{1}{10(\lambda_n+\lambda_p)} + V_{TH_n}$$

$$= \frac{1}{10(0.1+0.2)V^{-1}} + 0.4V \approx 0.73\ V$$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{TH_n})^2$$

$$= \frac{1}{2}\left(100\ \frac{uA}{V^2}\right)\left(\frac{20}{0.18}\right)(0.33V)^2 \approx 0.61\ mA$$

$$\because \text{power} = V_{DD}(I_{REF} + I_{D_1})$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - I_{D_1} = \frac{2mW}{1.8V} - 0.61mA$$

$$\approx 0.5\ mA$$

$\therefore$ if M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_{D2}}{I_{REF}} = \frac{61}{50} \approx 1.2$$

2. a) $V_1 = V_{ov1} + V_{TH}$

$I_{REF} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov1}^2 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov2}^2$, $V_{ov1} = V_{ov2}$

$\therefore V_2 = V_1 + V_{ov2} + V_{TH} = \mathbf{2(V_{ov1} + V_{TH})}$

b) $V_{ov1} = V_{ov3} = V_{ov2} = V_{ov4}$

To keep M3 in the saturation region, $V_{DS3} \geq V_1 - V_{TH}$

To keep M4 in the saturation region, $V_{out} \geq V_2 - V_{TH}$

Therefore, $\mathbf{V_{out} \geq 2V_{ov1} + V_{TH}}$

c) $V_1 = V_{ov1} + V_{TH}$

$I_{REF} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov1}^2 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov2}^2$, $V_{ov1} = V_{ov2}$

To keep M1 in saturation region, $V_{DS1} \geq V_{ov1}$

$V_2 = V_{DS1} + V_{ov2} + V_{TH}$

Therefore, $\mathbf{V_2 \geq 2V_{ov1} + V_{TH}}$

d) $V_2 \geq 2V_{ov1} + V_{TH}$, so $V_{out} \geq V_2 - V_{TH} \geq 2V_{ov1}$

$\therefore \mathbf{V_{out} \geq 2V_{ov1}}$

e) Figure 2 is better because it consumes less voltage headroom than Figure 1.

But Figure 2 need extra biasing circuit. It takes more power and area than Figure 1.

3. $V_{BE1} = V_{BE2} + I_{OUT} \times 1k\Omega$

$V_{BE1} = V_T \ln\left(\frac{1mA}{I_S}\right)$, $V_{BE2} = V_T \ln(\frac{I_{OUT}}{I_S})$

$V_{BE1} - V_{BE2} = 26mV \times \ln\left(\frac{1mA}{I_{OUT}}\right) = I_{OUT} \times 1k\Omega$

$\therefore I_{OUT} = 0.069mA$