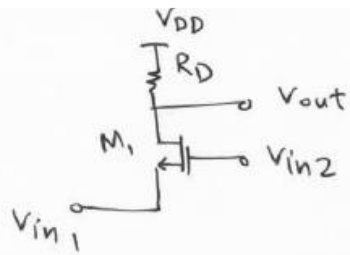


1.

10.30

(1)



$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{in2} - V_{in1} - V_{TH})^2$$

- (1) The current is not an odd function of $(V_{in2} - V_{in1})$. Therefore it is not symmetric around $V_{in1} = V_{in2}$ [$(V_{in1} - V_{in2}) = 0$].
- (2) The input impedance seen at V_{in1} and V_{in2} are different
- (3) The circuit cannot suppress the supply noise because there is no differential output available.

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D1}I_{D2}})$$

(a)

$$I_{D1} = 0 \Rightarrow$$

$$(V_{in1} - V_{in2})^2 = \frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum differential input voltage to turn M_1 off.

$$(b) \quad I_{D1} = \frac{I_{SS}}{2} \Rightarrow I_{D2} = \frac{I_{SS}}{2}$$

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - I_{SS}) = 0 \rightarrow V_{in1} - V_{in2} = 0$$

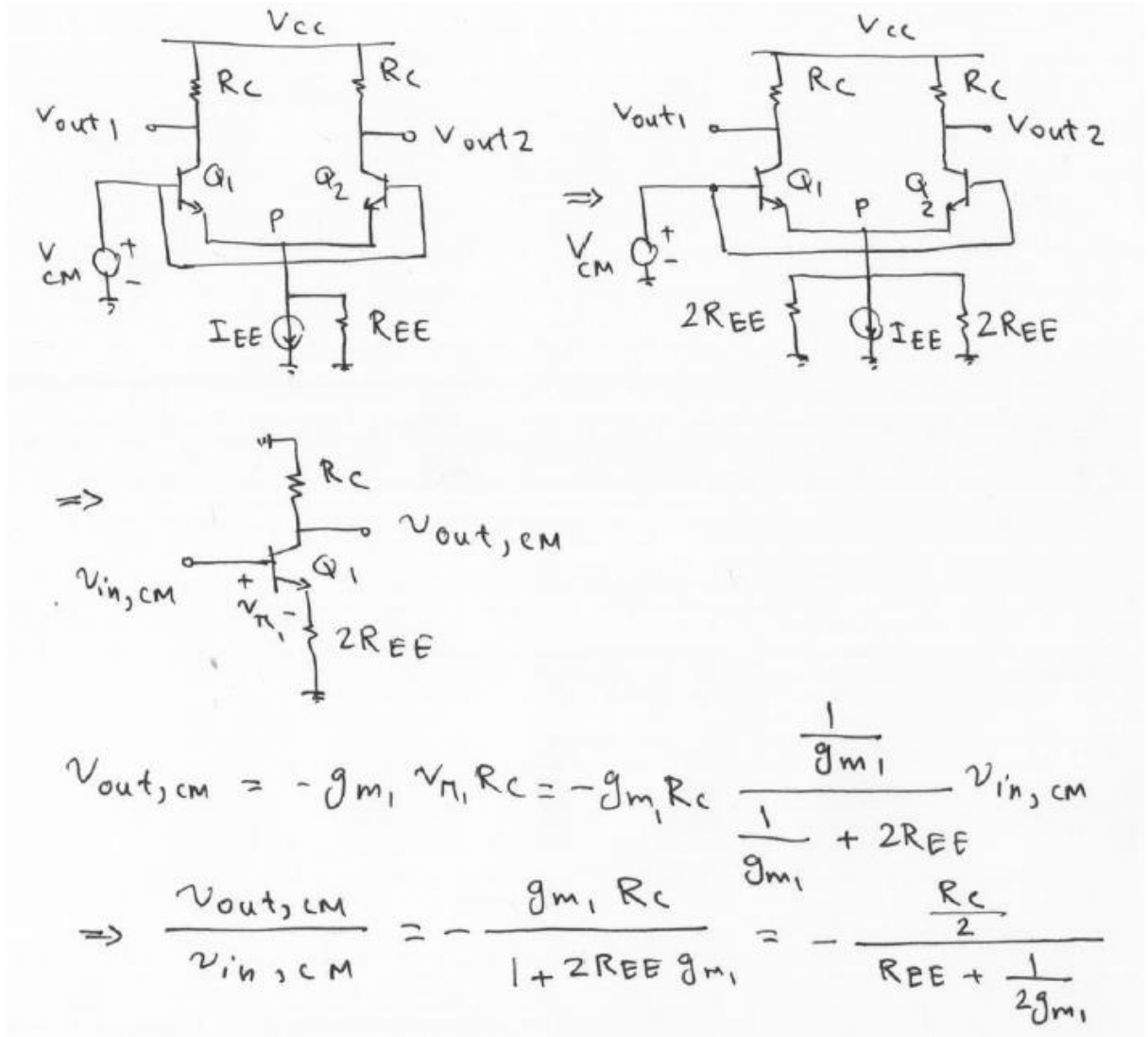
This is the equilibrium input case.

$$(c) \quad I_{D1} = I_{SS} \rightarrow I_{D2} = 0$$

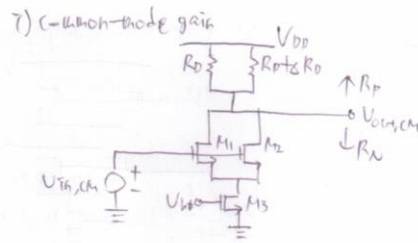
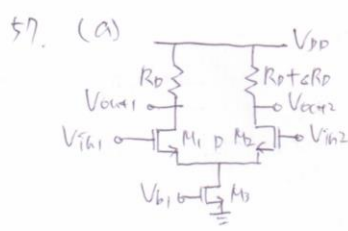
$$(V_{in1} - V_{in2})^2 = \frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in1} - V_{in2} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum input differential voltage to turn M_2 off.

10.53



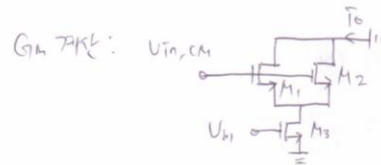
10.57
(a)



$$R_p = R_D \parallel (R_D + cR_D)$$

$$R_N = 2g_{m1} \frac{r_{o1}}{2} r_{o3} + \frac{r_{o1}}{2} + r_{o3} \approx g_{m1} r_{o1} r_{o3}$$

$$R_{out} = R_p \parallel R_N$$

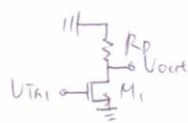


$$G_m = \frac{I_0}{V_{in,CM}} = \frac{2g_{m1} V_{gs1}}{V_{in,CM}}$$

$$= \frac{2g_{m1}}{V_{in,CM}} \frac{\frac{I}{2g_{m1}}}{\frac{I}{2g_{m1}} + r_{o3}} V_{in,CM} \approx \frac{I}{r_{o3}}$$

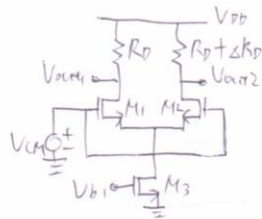
$$A_{CM} = -G_m R_{out}$$

ii) A_{DM} 은 계산하기 위하여 half circuit를 사용하오



$$A_{DM} = \frac{V_{out}}{V_{in1}} = -g_{m1} R_D$$

iii) A_{CM-DM} 계산



$$A_{CM-DM} = \frac{\Delta V_{out}}{\Delta V_{CM}}$$

$$\Delta V_{CM} = \Delta V_{gs} + 2\Delta I_D r_{o3}$$

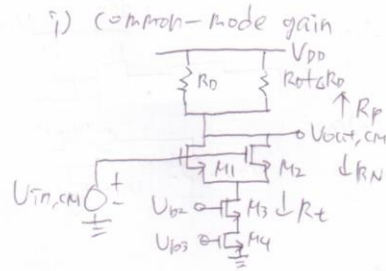
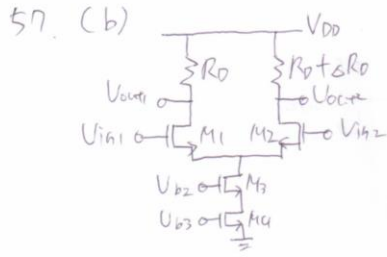
$$= \Delta I_D \left(\frac{1}{g_{m1}} + 2r_{o3} \right)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta R_D \Delta I_D$$

$$A_{CM-DM} = - \frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}$$

$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m1} R_D}{\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2r_{o3}}} = (1 + 2g_{m1} r_{o3}) \frac{R_D}{\Delta R_D}$$

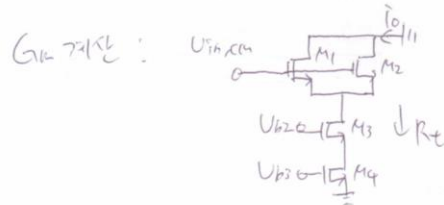
(b)



$$R_p = R_D \parallel (R_D + \Delta R_D)$$

$$R_N = \frac{r_{o1}}{2} + R_{te} + 2g_{m1} \frac{r_{o1}}{2} R_{te} \approx g_{m1} r_{o1} R_{te} \approx g_{m1} r_{o1} g_{m3} r_{o3} r_{o4}$$

$$R_{out} = R_p \parallel R_N = R_D \parallel (R_D + \Delta R_D) \parallel (g_{m1} r_{o1} g_{m3} r_{o3} r_{o4})$$



$$G_m = \frac{i_o}{V_{in,CM}} = \frac{2g_{m1} V_{gs1}}{V_{in,CM}}$$

$$= \frac{2g_{m1}}{V_{in,CM}} \frac{2g_{m1}}{2g_{m1}} V_{in,CM} \approx \frac{I}{R_{te}}$$

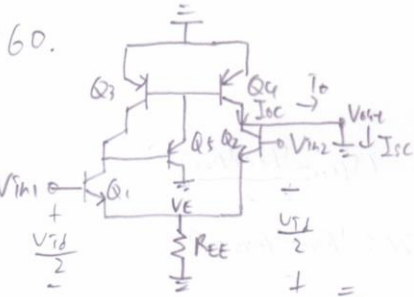
$$A_{CM} = -G_m R_{out} = -\frac{R_{out}}{R_{te}}$$

i) $A_{DM} = -g_{m1} R_D$ (57. (a)와 같음)

ii) $A_{CM-DM} = -\frac{\Delta R_D}{\frac{I}{g_{m1}} + 2[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}]}$

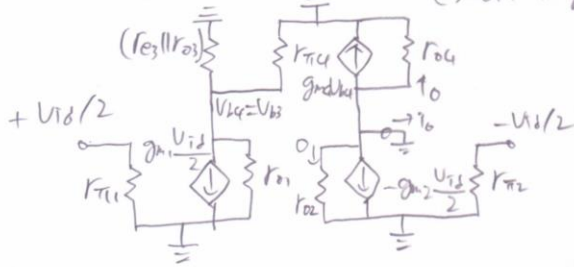
$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m1} R_D}{\frac{\Delta R_D}{\frac{I}{g_{m1}} + 2[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}]}} = \boxed{\left[1 + 2g_{m1} (g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}) \right] \times \frac{R_D}{\Delta R_D}}$$

(b)의 CMRR이 (a)의 CMRR보다 훨씬 크다.



Q3는 비역로 동작 → 생략가능

(small-signal)



$$V_{b3} = -g_{m1} \left(\frac{V_{id}}{2} \right) (r_{e3} || r_{o3} || r_{o1} || r_{\pi 4})$$

$$\approx -g_{m1} r_{e3} \left(\frac{V_{id}}{2} \right)$$

$$g_{m4} V_{b3} = -g_{m4} g_{m1} r_{e3} \left(\frac{V_{id}}{2} \right)$$

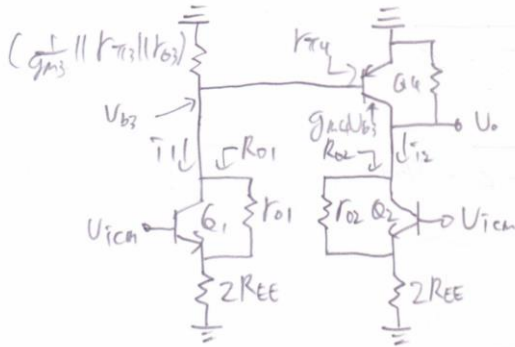
$$I_o = g_{m2} \left(\frac{V_{id}}{2} \right) - g_{m4} V_{b3}$$

$$= (g_{m2} + g_{m4} g_{m1} r_{e3}) \left(\frac{V_{id}}{2} \right)$$

$$G_m = I_o / V_{id} = (g_{m2} + g_{m4} g_{m1} r_{e3}) / 2 \approx g_{m1}$$

$$I_1 \approx I_2 \approx I_{cm} / 2 R_{EE}$$

(common-mode gain)



$$V_{b3} = -i_1 \left(\frac{1}{g_{m3}} || r_{\pi 3} || r_{o3} || r_{\pi 4} \right)$$

$$U_o = -r_{o4} (g_{m4} V_{b3} + i_2)$$

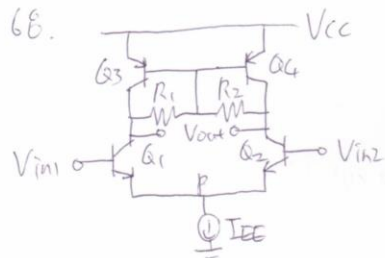
$$A_{cm} = \frac{U_o}{V_{icm}}$$

$$= \frac{r_{o4}}{2R_{EE}} \left[g_{m4} \left(\frac{1}{g_{m3}} || r_{\pi 3} || r_{o3} || r_{\pi 4} \right) - 1 \right]$$

$$\approx - \frac{r_{o4}}{2R_{EE}} \frac{\frac{2}{r_{\pi 3}}}{g_{m3} + \frac{2}{r_{\pi 3}}}$$

$$\approx - \frac{r_{o4}}{\beta_3 R_{EE}}$$

10.68

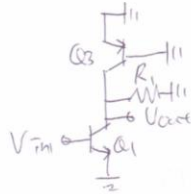


(Half Circuit)

$$A_v = 100$$

$$P = 1 \mu W$$

$$V_{A,n} = 10V, V_{A,p} = 5V, V_{CC} = 2.5V, R_1 = R_2$$



$$A_v = -g_{m1} (R_1 \parallel r_{o1} \parallel r_{o2})$$

$$g_{m1} = \frac{I_{EE}}{2V_T}, r_{o1} = \frac{2V_{A,n}}{I_{EE}}, r_{o2} = \frac{2V_{A,p}}{I_{EE}}$$

$$100 = \frac{1}{2V_T} \left[(R_1 \parallel I_{EE}) \parallel \underbrace{(2V_{A,n})}_{20} \parallel \underbrace{(2V_{A,p})}_{10} \right]$$

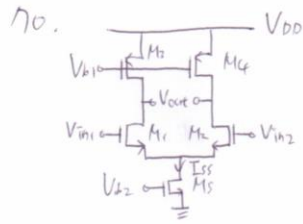
$$\frac{6.67 R_1 I_{EE}}{R_1 I_{EE} + 6.67} = 260 \times 260 = 5.2$$

$$6.67 R_1 I_{EE} = 5.2 R_1 I_{EE} + 5.2 \times 6.67$$

$$1.47 R_1 I_{EE} = 5.2 \times 6.67, R_1 I_{EE} = 2.359 \times 10$$

$$P = 1 \mu W = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4 \mu A$$

$$\Rightarrow R_1 = \frac{23.59}{I_{EE}} = 58.98 \text{ k}\Omega$$



$$A_v = 40$$

$$(V_{GS1} - V_{TH})_{\text{equil}} = ?$$

$$\lambda_n = 0.1 \text{ V}^{-1}, \lambda_p = 0.2 \text{ V}^{-1}$$

$$\mu_n C_{ox} = 100 \mu\text{A/V}^2, \mu_p C_{ox} = 100 \mu\text{A/V}^2$$

$$V_{DD} = 1.8 \text{ V}, P = 2 \text{ mW}$$

$$A_v = -g_m (r_{op} || r_{on}) = -\frac{I_{SS}}{(V_{GS1} - V_{TH})_{\text{equil}}} \left(\frac{1}{\frac{I_{SS}}{2} \lambda_n} || \frac{1}{\frac{I_{SS}}{2} \lambda_p} \right)$$

$$= -\frac{2}{(V_{GS1} - V_{TH})_{\text{equil}}} \left(\frac{1}{\lambda_n} || \frac{1}{\lambda_p} \right)$$

$$= \frac{2}{(V_{GS1} - V_{TH})_{\text{equil}}} (10 || 5) = 40 \Rightarrow (V_{GS1} - V_{TH})_{\text{equil}} = 166.67 \text{ mV}$$

$$P = 2 \times 10^{-3} = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{2 \times 10^{-3}}{1.8} = 1.11 \mu\text{A}$$

$$\left(\frac{W}{L} \right)_{1,2} = \frac{I_{SS}}{\mu_n C_{ox} (V_{GS1} - V_{TH})_{\text{equil}}^2} = \frac{1.11 \times 10^{-3}}{10^{-4} \times 0.16667^2} = 400$$

$$\left(\frac{W}{L} \right)_{3,4} = \frac{I_{SS}}{\mu_p C_{ox} (V_{GS1} - V_{TH})_{\text{equil}}^2} = \frac{1.11 \times 10^{-3}}{10^{-4} \times 0.16667^2} = 400$$

$$\left(\frac{W}{L} \right)_S = \frac{2I_{SS}}{\mu_n C_{ox} (V_{GS1} - V_{TH})_{\text{equil}}} = \frac{2 \times 1.11 \times 10^{-3}}{10^{-4} \times 0.16667} = 800$$

2.

$$(a) V_{in1} = V_{in2} \Rightarrow V_{offset} = \Delta R \times \frac{1}{2} I_{SS}$$

$$(b) V_{in1} = V_{in2} \quad \Delta I_D = \frac{1}{2} \mu_n C_{ox} \Delta \left(\frac{W}{L} \right) (V_{in} - V_{SS} - V_{th})^2$$

$$I_{SS} = 2 \times \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{in} - V_{SS} - V_{th})^2$$

$$\Rightarrow \Delta I_D = I_{SS} \times \frac{\Delta \left(\frac{W}{L} \right)}{2 \left(\frac{W}{L} \right)}$$

$$\therefore V_{offset} = R \cdot \Delta I_D = R I_{SS} \cdot \frac{\Delta \left(\frac{W}{L} \right)}{2 \left(\frac{W}{L} \right)}$$

$$(c) \Delta \left(\frac{W}{L} \right) \neq 0, \Delta R \neq 0.$$

when $V_{in1} = V_{in2}$,

$$V_{out1} - V_{out2} = \Delta R \cdot \frac{1}{2} I_{SS} + R \cdot \Delta I_D \quad (\text{1차 근사})$$

$$\Rightarrow \Delta R \cdot \frac{1}{2} I_{SS} + R \cdot \frac{1}{2} I_{SS} \cdot \frac{\Delta \left(\frac{W}{L} \right)}{\left(\frac{W}{L} \right)}$$

$$= \frac{1}{2} I_{SS} \left(\Delta R + R \cdot \frac{\Delta \left(\frac{W}{L} \right)}{\left(\frac{W}{L} \right)} \right)$$

$$V_{in, offset} = \frac{V_{out, offset}}{A_v}$$

$$= \frac{1}{2} I_{SS} / g_m \cdot \left[\frac{\Delta R}{R} + \frac{\Delta \left(\frac{W}{L} \right)}{\left(\frac{W}{L} \right)} \right]$$