Homework #2 - SOLUTIONS

- * Answer the following questions. Be sure to clearly show the procedures to solve the problems.
- 1. You want to develop an eco-friendly and cost-effective process for removal of nitrate (NO_3) from groundwater. Your plan is to supply molasses, a byproduct of sugar manufacturing, as an e⁻ donor to enhance denitrification in groundwater. Assuming that the molecular formular of molasses can be represented by $C_{12}H_{22}O_{11}$ (same as that for sugar), answer the following.
- 1) Write the electron donor half reaction, R_d, in an electron-equivalent form. Use HCO3⁻ as an only form of an oxidized carbon species. (20 points)

Step 1:
$$HCO_3^- = C_{12}H_{22}O_{11}$$

Step 2: $HCO_3^- + H_2O + e^- = C_{12}H_{22}O_{11}$
Step 3: $12HCO_3^- + H_2O + e^- = C_{12}H_{22}O_{11}$
Step 4: $12HCO_3^- + e^- = C_{12}H_{22}O_{11} + 25H_2O$
Step 5: $12HCO_3^- + 60H^+ + e^- = C_{12}H_{22}O_{11} + 25H_2O$
Step 6: $12HCO_3^- + 60H^+ + 48e^- = C_{12}H_{22}O_{11} + 25H_2O$
Step 7: $\frac{1}{4}HCO_3^- + \frac{5}{4}H^+ + e^- = \frac{1}{48}C_{12}H_{22}O_{11} + \frac{25}{48}H_2O$

2) Write the energy reaction, R_e, in an electron-equivalent form. How much grams of molasses are needed per g of NO3-N consumed for the energy reaction? (20 points)

Solution)

Solution)

$$R_a (I-7): \frac{1}{5}NO_3^- + \frac{6}{5}H^+ + e^- = \frac{1}{10}N_2 + \frac{3}{5}H_2O$$
$$-R_d: \frac{1}{48}C_{12}H_{22}O_{11} + \frac{25}{48}H_2O = \frac{1}{4}HCO_3^- + \frac{5}{4}H^+ + e^-$$

$$R_e: \qquad \frac{1}{48} C_{12}H_{22}O_{11} + \frac{1}{5} NO_3^- = \frac{1}{10} N_2 + \frac{1}{4} HCO_3^- + \frac{1}{20} H^+ + \frac{19}{240} H_2O$$

Molasses molecular weight: $12 \times 12 + 1 \times 22 + 16 \times 11 = 342$

g molasses needed/g NO₃-N consumed =
$$-\frac{1}{48}$$
 mole×342 g molasses/mole = 2.54 $-\frac{1}{5}$ mole×14 g NO₃-N/mole

3) Write the cell formation half reaction, R_c , in an electron-equivalent form. Use the cell formula of $C_5H_7O_2N$ and NO_3^- as a source of nitrogen (not NH_4^+). Also use HCO_3^- as an only form of an oxidized carbon species. (20 points)

Solution)

Step 1:
$$HCO_{3}^{-} = C_{5}H_{7}O_{2}N$$

Step 2: $HCO_{3}^{-} + NO_{3}^{-} + H_{2}O + e^{-} = C_{5}H_{7}O_{2}N$
Step 3: $5HCO_{3}^{-} + NO_{3}^{-} + H_{2}O + e^{-} = C_{5}H_{7}O_{2}N$
Step 4: $5HCO_{3}^{-} + NO_{3}^{-} + e^{-} = C_{5}H_{7}O_{2}N + 16H_{2}O$
Step 5: $5HCO_{3}^{-} + NO_{3}^{-} + 34H^{+} + e^{-} = C_{5}H_{7}O_{2}N + 16H_{2}O$
Step 6: $5HCO_{3}^{-} + NO_{3}^{-} + 34H^{+} + 28e^{-} = C_{5}H_{7}O_{2}N + 16H_{2}O$
Step 7: $\frac{5}{28}HCO_{3}^{-} + \frac{1}{28}NO_{3}^{-} + \frac{17}{14}H^{+} + e^{-} = \frac{1}{28}C_{5}H_{7}O_{2}N + \frac{4}{7}H_{2}O$

4) Write the overall cell synthesis reaction, R_s , in an electron-equivalent form. Use the R_d derived from 1) and the R_c derived from 3). How much grams of molasses are needed per g of NO₃-N consumed for the cell synthesis reaction? (20 points)

Solution)

$$R_{c}: \frac{5}{28}HCO_{3}^{-} + \frac{1}{28}NO_{3}^{-} + \frac{17}{14}H^{+} + e^{-} = \frac{1}{28}C_{5}H_{7}O_{2}N + \frac{4}{7}H_{2}O$$

- $R_{d}: \frac{1}{48}C_{12}H_{22}O_{11} + \frac{25}{48}H_{2}O = \frac{1}{4}HCO_{3}^{-} + \frac{5}{4}H^{+} + e^{-}$
- $R_{s}: \frac{1}{48}C_{12}H_{22}O_{11} + \frac{1}{28}NO_{3}^{-} = \frac{1}{28}C_{5}H_{7}O_{2}N + \frac{1}{14}HCO_{3}^{-} + \frac{1}{28}H^{+} + \frac{17}{336}H_{2}O$

g molasses needed/g NO₃-N consumed =
$$\begin{bmatrix} -\frac{1}{48} & mole \times 342 & g & molasses/mole \\ -\frac{1}{28} & mole \times 14 & g & NO_3 - N/mole \end{bmatrix} = 14.25$$

5) From the calculations you did for 2) and 4), which growth state do you think is more favorable for efficient use of molasses? (A) a rapidly growing state or (B) a slowly growing state? Briefly describe the reason for your selection. (10 points)

Solution)

<u>(B)</u>

At a slowly growing state, the f_e value is greater, meaning that the overall stoichiometry is more weighted to R_e than it is for a slowly growing state. Therefore, at a slowly growing state the amount of molasses needed to remove a gram of NO₃-N will be greater (more efficient use of molasses).

6) You are planning to control the molasses supply rate and other environmental conditions relevant to the bacterial growth such that a f_s value of 0.05 is achieved. Write the stoichiometry of the overall reaction occurring at this condition. For 1 g of NO₃-N consumption, i) how much molasses will be consumed (in g molasses), ii) how much alkalinity will be produced (in g as CaCO₃), and iii) how much biomass will be produced (in g biomass)? (30 points)

Solution)

 $f_s = 0.05, f_e = 0.95$

 $f_s R_s \stackrel{.}{.} 0.001042 C_{\!12} H_{\!22} O_{\!11} + 0.001786 NO_{\!3}^{-}$

 $= 0.001786 C_5 H_7 O_2 N + 0.003571 H CO_3^- + 0.001786 H^+ + 0.002530 H_2 O_3^- + 0.001786 H^+ + 0.001$

 $f_e R_e \stackrel{.}{.} 0.01979 C_{12} H_{22} O_{11} + 0.19 N O_3^- = 0.095 N_2 + 0.2375 H C O_3^- + 0.0475 H^+ + 0.07521 H_2 O_3^- + 0.0475 H^- + 0.07521 H_2 O_3^- + 0.01979 H_2^- + 0.01975 H_2^-$

 $R: 0.02083C_{12}H_{22}O_{11} + 0.1918NO_3^{-1}$

 $= 0.001786 C_5 H_7 O_2 N + 0.2411 H CO_3^- + 0.095 N_2 + 0.04929 H^+ + 0.07774 H_2 O_3^- + 0.095 N_2 +$

i) molasses consumption

 $\frac{0.02083\ mole \times 342\ g\ molasses/mole}{0.1918\ mole \times 14\ g\ NO_3 - N/mole} = 2.65\ g\ molasses/g\ NO_3 - N$

ii) alkalinity production

 HCO_3^- : alkalinity-contributing species; H^+ : alkalinity-consuming species

Total alkalinity produced = 0.2411 - 0.04929 = 0.1918 equivalent (for 1 e-equivalent reaction) Per g NO₃-N basis, 0.1918 eq×50 g as CaCO₃ $\overline{0.1918 \text{ mole} \times 14 \text{ g NO}_3 - N/\text{mole}} = 3.57 \text{ g as } CaCO_3/\text{g NO}_3 - N$

iii) biomass production

 $\frac{0.001786\ mole \times 113\ g\ cells/mole}{1918\ mole \times 14\ g\ NO_3 - N/mole} = 0.0752\ g\ cells/g\ NO_3 - N$

2. At the following conditions, at what O₂ partial pressure will an energy reaction using ferric ion (Fe³⁺) as an e⁻ acceptor be more thermodynamically favorable than an energy reaction using O₂ as an e⁻ acceptor? From your results, briefly discuss about the condition where ferric ion reduction will be favorable.

pH = 8.0 $T=20^{\circ}C$ $[Fe^{2+}] = 8.0 \times 10^{-4} M$ (dissolved at the solubility limit of Fe(OH)₂ at pH=8.0) $[Fe^{3+}] = 4.0 \times 10^{-20} M$ (dissolved at the solubility limit of Fe(OH)₃ at pH=8.0) (40 points) Solution) Ferrous-Ferric half-reaction: $Fe^{3+} + e^- = Fe^{2+}$. $\Delta G_{r}^{0'} = -74.27 \ kJ/e^{-} \ eq$ $\Delta G_r^0 = \Delta G_r^{0'} - RTv_{H^+} \ln[10^{-7}] = \Delta G_r^{0'} = -74.27 \ kJ/e^- \ eq$ $\Delta G_{r} = \Delta G_{r}^{0} + RT \ln \frac{[Fe^{2+}]}{[Fe^{3+}]} = -74.27 + 8.314 \times 10^{-3} \ kJ/mole - K \times 293 \ K \times \ln \frac{8.0 \times 10^{-4}}{4.0 \times 10^{-20}}$ $= 17.16 \ kJ/e^{-} \ eq$ Water-Oxygen half-reaction: $\frac{1}{4}O_2 + H^+ + e^- = \frac{1}{2}H_2O$ $\Delta \textit{G}_{r}^{0} = \Delta \textit{G}_{r}^{0\prime} - \textit{RTv}_{\textit{H}^{+}} \ln [10^{-7}] = -78.72 - 8.314 \times 10^{-3} \textit{ kJ/mole} - \textit{K} \times 298 \textit{ K} \times (-1) \times \ln [10^{-7}]$ $(\Delta G_r^0 \text{ is at } 25^{\circ}C)$ $=-118.65 \ kJ/e^{-} \ eq$ $\Delta G_{r} = \Delta G_{r}^{0} + RT \ln \frac{1}{P_{Q_{r}}^{1/4} \cdot [H^{+}]} = -118.65 + 8.314 \times 10^{-3} \, kJ / mole - K \times 293 \, K \times \ln \frac{1}{P_{Q_{r}}^{1/4} \cdot 10^{-8.0}}$ $= 17.16 \ kJ/e^{-} \ eq$

$$\begin{split} \Delta G_{\!r} &= \Delta G_{\!r}^0 + RT {\rm ln} - \frac{1}{P_{O_2}^{1/4}} \cdot \left[H^+ \right] = & -118.65 + 44.87 + 8.314 \times 10^{-3} \; kJ\!/mole - K\!\times 293 \; K\!\times \frac{1}{4} \, {\rm ln} - \frac{1}{P_{O_2}} \\ &= 17.16 \; kJ\!/e^- \; eq \end{split}$$

$$-\ln P_{O_2} = 149.33$$

$$P_{O_2} = 1.40 \times 10^{-65} atm$$

The results suggest that under any meaningful oxygen partial pressure, aerobic process will be more thermodynamically favorable than iron reduction.