

1.

11-1

11.1

$$\cdot \text{power} = 2.5\text{mW} = VI = 2.5 \times I_c$$

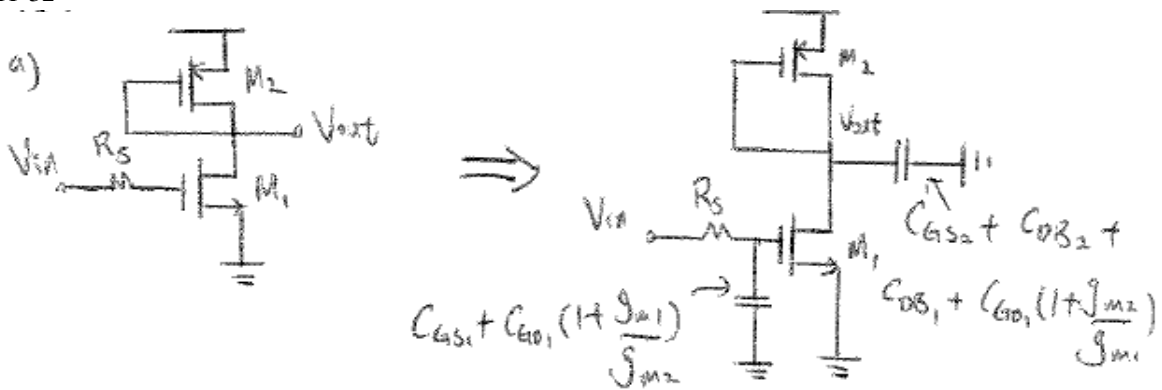
$$\rightarrow I_c = 1\text{mA}$$

$$\cdot \text{max. DC gain} = 1.0 = \left| -\frac{I_c \cdot R_1}{V_T} \right| = \frac{1\text{mA} \cdot R_1}{26\text{mV}}$$

$$\rightarrow R_1 = 46.8\Omega$$

$$\therefore \frac{1}{2\pi \times R_1 \times C_L} = 1.2\text{GHz}$$

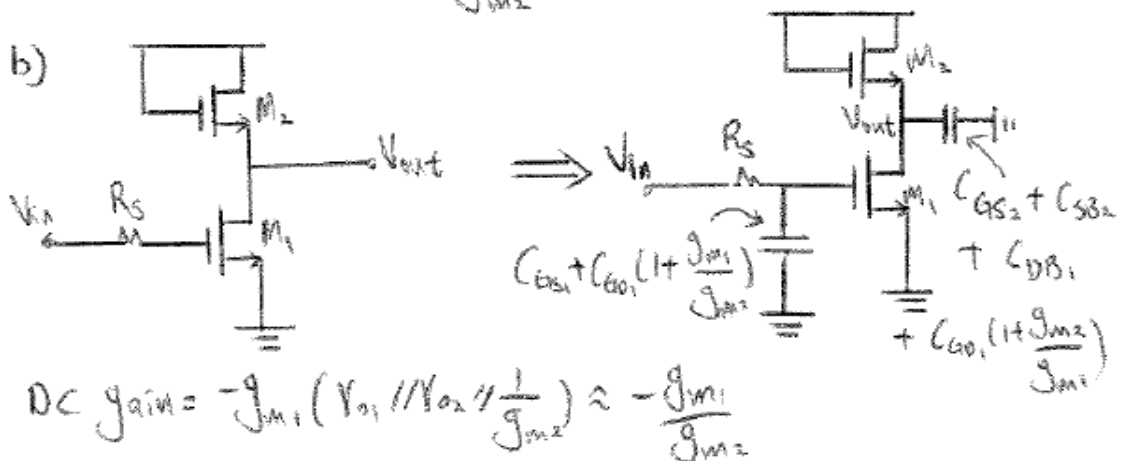
$$\rightarrow C_L \approx 2.84\text{pF}$$



$$DC \text{ gain} = -g_{m1} \left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \right) \approx -\frac{g_{m1}}{g_{m2}}$$

$$\omega_{p_{in}} = \frac{1}{R_s \left(C_{GS1} + C_{GD1} \left(1 + \frac{g_{m1}}{g_{m2}}\right) \right)}$$

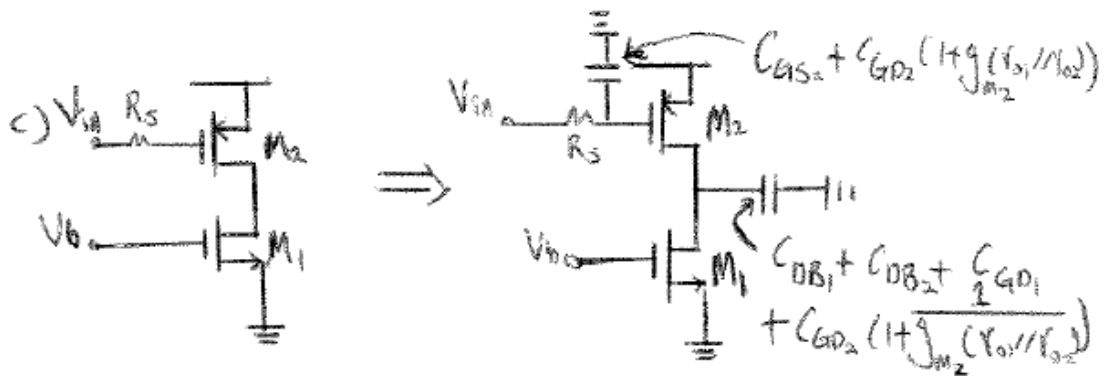
$$\omega_{p_{out}} = \frac{g_{m2}}{C_{GS2} + C_{DB2} + C_{DB1} + C_{GD1} \left(1 + \frac{g_{m2}}{g_{m1}}\right)}$$



$$DC \text{ gain} = -g_{m1} \left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \right) \approx -\frac{g_{m1}}{g_{m2}}$$

$$\omega_{p_{in}} = \frac{1}{R_s \left(C_{GS1} + C_{GD1} \left(1 + \frac{g_{m1}}{g_{m2}}\right) \right)}$$

$$\omega_{p_{out}} = \frac{g_{m2}}{C_{SB2} + C_{GS2} + C_{DB1} + C_{GD1} \left(1 + \frac{g_{m2}}{g_{m1}}\right)}$$



DC gain: $-g_{m2} (V_{o1} // V_{o2})$

$$\omega_{pin} = \frac{1}{R_S (C_{GS2} + C_{GD2} (1 + g_{m2} (V_{o1} // V_{o2})))}$$

$$\omega_{pout} = \frac{1}{(V_{o1} // V_{o2}) [C_{DB1} + C_{DB2} + C_{GD1} + C_{GD2} (1 + \frac{1}{g_{m2} (V_{o1} // V_{o2})})]}$$

$$\omega_{pout} \approx \frac{1}{(V_{o1} // V_{o2}) [C_{DB1} + C_{DB2} + C_{GD1} + C_{GD2}]}$$

Since $g_{m2} (V_{o1} // V_{o2}) \gg 1$

(a) Note that the DC gain is $A_v = -\infty$ if we assume $V_A = \infty$.

$$\omega_{p,in} = \frac{1}{(R_S \parallel r_\pi) [C_\pi + C_\mu (1 - A_v)]} = \boxed{0}$$

$$\omega_{p,out} = \boxed{0}$$

(b)

$$\frac{V_{out}}{V_{Thev}}(s) = \lim_{R_L \rightarrow \infty} \frac{(C_\mu s - g_m) R_L}{as^2 + bs + 1}$$

$$a = (R_S \parallel r_\pi) R_L (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})$$

$$b = (1 + g_m R_L) C_\mu (R_S \parallel r_\pi) + (R_S \parallel r_\pi) C_\pi + R_L (C_\mu + C_{CS})$$

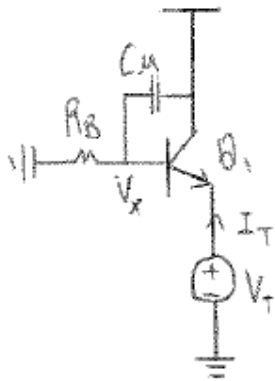
$$\lim_{R_L \rightarrow \infty} \frac{(C_\mu s - g_m) R_L}{as^2 + bs + 1} = \frac{C_\mu s - g_m}{[(R_S \parallel r_\pi) (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})] s^2 + [g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}] s}$$

$$= \frac{C_\mu s - g_m}{s \{ (R_S \parallel r_\pi) (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}) s + [g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}] \}}$$

$$|\omega_{p1}| = \boxed{0}$$

$$|\omega_{p2}| = \boxed{\frac{g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}}{(R_S \parallel r_\pi) (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}}$$

We can see that the Miller approximation correctly predicts the input pole to be at DC. However, it incorrectly estimates the output pole to be at DC as well, when in fact it is not, as we can see from the direct analysis.



$$V_A = \infty$$

$$\frac{\beta}{\beta+1} \approx 1, \text{ if } \beta \gg 1$$

$$I_T = -(V_x - V_T) g_m \approx -(V_x - V_T) g_m$$

$$V_x = \frac{I_T}{\beta} \left(R_B \parallel \frac{1}{C_u s} \right)$$

$$I_T = \left(V_T - \frac{I_T}{\beta} \left(R_B \parallel \frac{1}{C_u s} \right) \right) g_m$$

$$I_T = g_m V_T - \frac{g_m}{\beta} \left(R_B \parallel \frac{1}{C_u s} \right) I_T$$

$$I_T \left(1 + \frac{g_m}{\beta} \left(R_B \parallel \frac{1}{C_u s} \right) \right) = g_m V_T$$

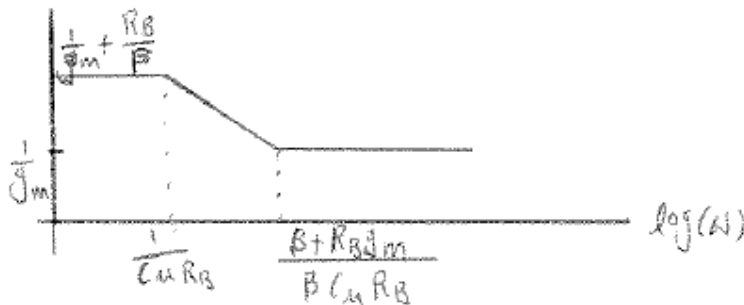
$$\frac{V_T}{I_T} = \frac{1}{g_m} + \frac{R_B \parallel \frac{1}{C_u s}}{\beta} = \frac{\beta C_u R_B (s + \frac{\beta + R_B g_m}{\beta C_u R_B})}{g_m \beta (1 + C_u R_B s)}$$

$$\text{zero: } \frac{\beta + R_B g_m}{\beta C_u R_B}, \quad \text{pole: } \frac{1}{C_u R_B}$$

$$\text{At DC, } |Z_{out}| = \frac{1}{g_m} + \frac{R_B}{\beta}$$

$$\text{At very high freq: } |Z_{out}| = \frac{1}{g_m}$$

20 log |Zout|



$$I_D = \frac{1}{2} \left(\frac{W}{L} \right)_1 \mu_n C_{ox} V_{ov}^2 = 0.5 \text{ mA}$$

$$(W/L)_1 = (W/L)_2 = \boxed{250}$$

$$W_1 = W_2 = 45 \text{ } \mu\text{m}$$

$$g_{m1} = g_{m2} = \frac{W}{L} \mu_n C_{ox} V_{ov} = 5 \text{ mS}$$

$$C_{GD1} = C_{GD2} = C_0 W = 9 \text{ fF}$$

$$C_{GS1} = C_{GS2} = \frac{2}{3} W L C_{ox} = 64.8 \text{ fF}$$

$$\omega_{p,in} = \frac{1}{R_G \left\{ C_{GS1} + C_{GD1} \left(1 + \frac{g_{m1}}{g_{m2}} \right) \right\}} = 2\pi \times 5 \text{ GHz}$$

$$R_G = \boxed{384 \text{ } \Omega}$$

$$\omega_{p,out} = \frac{1}{R_D C_{GD2}} = 2\pi \times 10 \text{ GHz}$$

$$R_D = \boxed{1.768 \text{ k}\Omega}$$

$$A_v = -g_{m1} R_D = \boxed{-8.84}$$

2.1

$$A_v = -g_m \cdot R_L = \frac{1}{150} \times 2k\Omega \approx 13.3V/V$$

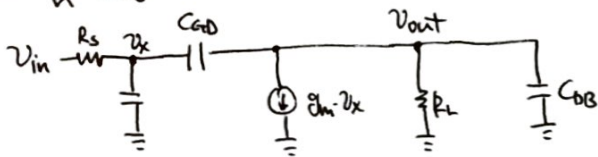
① input side pole

$$|\omega_1| = \frac{1}{R_s \times (C_{GS} + (1 + g_m R_L) \cdot C_{GD})} \approx 3.59 \text{ Grad/s} \quad \therefore \underline{\underline{f_{p1} = 571.4 \text{ MHz}}}$$

② output side pole

$$|\omega_2| = \frac{1}{R_L \times (C_{DB} + (1 + \frac{1}{g_m R_L}) \cdot C_{GD})} \approx 2.69 \text{ Grad/s} \quad \therefore \underline{\underline{f_{p2} = 417.8 \text{ MHz}}}$$

2.2 & 2.3



① V_x side

$$\frac{V_{in} - V_x}{R_s} = sC_{GS} \cdot V_x + sC_{GD} \cdot (V_x - V_{out})$$

② V_{out} side

$$g_m V_x + \frac{V_{out}}{R_L} + sC_{DB} \cdot V_{out} = sC_{GD} (V_x - V_{out})$$

→ 정리하면

$$\frac{V_{out}}{V_{in}} = \frac{(sC_{GD} - g_m)R_L}{as^2 + bs + 1}$$

$$\text{where, } \begin{cases} a = R_s \cdot R_L \cdot (C_{GS} \cdot C_{GD} + C_{DB} \cdot C_{GD} + C_{GS} \cdot C_{DB}) \\ b = (1 + g_m R_L) C_{GD} \cdot R_s + R_s \cdot C_{GS} + R_L (C_{GD} + C_{DB}) \end{cases}$$

⇒ "exact solution"

if $\omega_{p2} \gg \omega_{p1}$,

$$|\omega_{p1}| = \frac{1}{b}, \quad |\omega_{p2}| = \frac{b}{a}, \quad |\omega_z| = \frac{g_m}{C_{GD}} \quad \Rightarrow \underline{\underline{\text{"dominant-pole approximation"}}$$

각각 대입하면 계산하면

2.2 dominant-pole approximation

$$\left(\begin{array}{l} |\omega_{p1}| \approx 1.564 \text{ Grad/s}, \quad \therefore f_{p1} = 249 \text{ MHz} \\ |\omega_{p2}| \approx 30.157 \text{ Grad/s}, \quad \therefore f_{p2} = 4.80 \text{ GHz} \\ |\omega_z| \approx 83.333 \text{ Grad/s}, \quad \therefore f_z = 13.263 \text{ GHz} \end{array} \right.$$

2.3 exact location

$$\left(\begin{array}{l} |\omega_{p1}| \approx 1.654 \text{ Grad/s} \quad \therefore f_{p1} = 263 \text{ MHz} \\ |\omega_{p2}| \approx 20.502 \text{ Grad/s} \quad \therefore f_{p2} = 4.536 \text{ GHz} \\ |\omega_z| \approx 83.333 \text{ Grad/s} \quad \therefore f_z = 13.263 \text{ GHz} \end{array} \right.$$

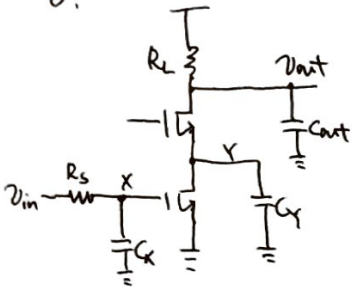
2.4

Miller effect 가 부정확한 이유

- ① 낮은 주파수의 gain (DC gain) 으로 근사한 것이나 주파수가 높아질수록 부정확
- ② zero 가 반영안됨

dominant-pole approximation의 경우 $f_{p2} > 10 \cdot f_{p1}$ 이고, 근사값과 exact solution이 거의 유사하다.

3.



$$\frac{V_y}{V_x} = -\frac{g_{m1}}{g_{m2}}$$

$$C_x = C_{GS1} + C_{GD1} \left(1 + \frac{g_{m1}}{g_{m2}}\right)$$

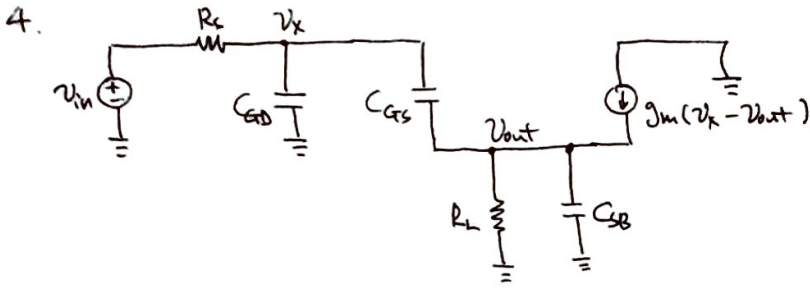
$$C_y = C_{GS2} + C_{GD1} \left(1 + \frac{g_{m2}}{g_{m1}}\right) + C_{DB1} + C_{SB2}$$

$$C_{out} = C_{DB2} + C_{GD2}$$

따라서 pole은

- ① $|\omega_{p,x}| = \frac{1}{C_x \cdot R_s} \approx 12.195 \text{ Grad/s} \quad , \quad f_{p,x} = 1.94 \text{ GHz}$
- ② $|\omega_{p,y}| = \frac{1}{C_y \cdot \frac{1}{g_{m2}}} \approx 10.929 \text{ Grad/s} \quad , \quad f_{p,y} = 1.74 \text{ GHz}$
- ③ $|\omega_{p,out}| = \frac{1}{C_{out} \cdot R_L} \approx 2.778 \text{ Grad/s} \quad , \quad f_{p,out} = 442 \text{ MHz}$

이들의 exact location의 1st pole 263MHz는 대비 약 2배정도 1st pole frequency가 증가한다. 즉 bandwidth 증가



① v_x 스텝

$$\frac{v_{in} - v_x}{R_s} = s \cdot C_{GD} \cdot v_x + s \cdot C_{GS} \cdot (v_x - v_{out})$$

② v_{out} 스텝

$$s \cdot C_{GS} \cdot (v_x - v_{out}) = \frac{1}{R_L} v_{out} + s C_{SB} \cdot v_{out} - g_m (v_x - v_{out})$$

→ 두 식을 정리하면

$$\frac{v_{out}}{v_{in}} = \frac{1 + s \frac{C_{GS}}{g_m}}{as^2 + bs + c}$$

where

$$a = \frac{R_s}{g_m} (C_{GD} \cdot C_{GS} + C_{GD} \cdot C_{SB} + C_{GS} \cdot C_{SB})$$

$$b = \frac{R_s}{g_m} (C_{GD} + C_{GS}) \left(g_m + \frac{1}{R_L} \right) + \frac{C_{GS} + C_{SB}}{g_m} - C_{GS} \cdot R_s$$

$$c = 1 + \frac{1}{g_m R_L}$$

$$\therefore \omega_{p1} = 2\pi \left[-3.606 \text{ GHz} + j(1.901 \text{ GHz}) \right]$$

$$\omega_{p2} = 2\pi \left[-3.606 \text{ GHz} - j(1.901 \text{ GHz}) \right]$$