Homework #4

*** Answer the following questions. Be sure to clearly show the procedures to solve the problems.**

A chemostat having V = 2,000 m^3 receives a flow rate of Q = 1,000 m^3/d of wastewater containing S^0 = 500 mg BOD_L/L. Also included in the wastewater is the inert biomass X_i^0 = 50 mg VSS/L. The following parameters are found for aerobic biodegradation:

$\hat{q} = 20\ g\ BOD_L/g\ VSS_a - d$	$k_2 = 0.09\ g\ COD_p/g\ VSS_a - d$
$Y = 0.42\ g\ VSS_a/g\ BOD_L$	$\hat{q}_{UAP} = 1.8\ g\ COD_p/g\ VSS_a - d$
$K = 20\ mg\ BOD_L/L$	$K_{UAP} = 100\ mg\ COD_p/L$
$b = 0.15/d$	$\hat{q}_{BAP} = 0.1\ g\ COD_p/g\ VSS_a - d$
$f_d = 0.8$	$K_{BAP} = 85\ mg\ COD_p/L$
$k_1 = 0.12\ g\ COD_p/g\ BOD_L$	

1. Calculate $S_{\min}$, $\theta_x^{\min}$ and θ_x of the chemostat. (10 points)

2. Calculate effluent VSS, COD and BOD_L. (30 points)

3. Calculate the effluent N and P concentrations when influent concentrations are 50 mg NH_4-N/L and 10 mg PO_4-P/L, respectively. (20 points)

4. Calculate the amount of O_2 that should be supplied to the reactor when influent and effluent DO are 6 and 2 mg/L, respectively. (20 points)

5. Assuming that the influent also contains biodegradable particulate organic matter with a concentration of 100 mg COD/L and the hydrolysis rate coefficient is k_{hyd} = 0.2/d, recalculate the effluent VSS, COD, and BOD_L. (30 points)

Hints:

- Effluent VSS should include both active and inert biomass, and particulate organic matter supplied from the influent (if there is any)
  (X_v = X_a + X_i + S_p in VSS)
  (for COD→VSS conversion of S_p, assume S_p has a chemical formula similar to biomass)

- Effluent COD should include COD of the substrate, SMP, and VSS
  (eff. COD = substrate COD + SMP COD + VSS COD)
  · Conversion needed for VSS: recall 1.42 g COD/g VSS for biomass ($C_5H_7O_2N$)

- BOD_L stands for "ultimate BOD", the oxygen demand for all biodegradable organic matter
  · S^0 is given as "BOD_L/L", so substrate is assumed to be fully biodegradable
  · SMP is fully biodegradable
  · active biomass is partially biodegradable (biodegradable fraction = f_d)
  · inert biomass is non-biodegradable
  So: eff. BOD_L
  = substrate BOD_L (=COD) + SMP BOD_L (=COD) + f_d × active biomass COD

Solution)

1) $S_{\min}$, $\theta_x^{\min}$ *and* θ_x

$$S_{\min} = K\frac{b}{Y\hat{q}-b} = (20\ mg\ BOD_L/L)\frac{0.15/d}{(0.42\ g\ VSS_a/g\ BOD_L)(20\ g\ BOD_L/g\ VSS_a-d)-0.15/d}$$
$$=0.36\ mg\ BOD_L/L$$

$$\theta_x^{\min} = \frac{K+S^0}{S^0(Y\hat{q}-b)-bK} = \frac{(20+500)\ mg\ BOD_L/L}{(500\ mg\ BOD_L/L)(0.42\cdot 20/d-0.15/d)-(20\ mg\ BOD_L/L)(0.15/d)}$$
$$=0.126\ d$$

$$\theta_x = \theta = \frac{V}{Q} = \frac{2000m^3}{1000\ m^3/d} = 2\ d$$

$$\frac{\theta_x}{\theta_x^{\min}} = SF = \frac{2\ d}{0.126\ d} = 16$$ *(SF of 16 for washout)*

2) Effluent VSS, COD, BOD$_L$

Firstly, we need to determine the effluent substrate and active biomass concentrations:

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x - (1+b\theta_x)}$$

$$= (20\ mg\ BOD_L/L)\frac{1+(0.15/d)(2\ d)}{(0.42\ g\ VSS_a/g\ BOD_L)(20\ g\ BOD_L/g\ VSS_a - d)(2\ d) - (1+(0.15/d)(2\ d))}$$

$$= 1.7\ mg\ BOD_L/L$$

$$X_a = Y(S^0 - S)\frac{1}{1+b\theta_x} = (0.42\ g\ VSS_a/g\ BOD_L)(500 - 1.7\ mg\ BOD_L/L)\frac{1}{1+(0.15/d)(2\ d)}$$

$$= 161\ mg\ VSS_a/L$$

Calculate the effluent inert VSS concentration:

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x$$

$$= 50\ mg\ VSS_i/L + (161\ mg\ VSS_a/L)(1-0.8g\ VSS_i/g\ VSS_a)(0.15/d)(2\ d)$$

$$= 60\ mg\ VSS_i/L$$

Now,

$$X_v = X_a + X_i = 161 + 60 = 221\ mg\ VSS/L$$

Think about the composition of effluent COD & BOD$_L$:

Effluent COD = remaining substrate + SMP + all VSS (active biomass + inert)

Effluent BOD$_L$ = remaining substrate + SMP + active and biodegradable biomass

** Effluent bsCOD = Effluent sBOD$_L$ = remaining substrate + SMP*

Effluent bpCOD = all VSS

Effluent pBOD$_L$ = active and biodegradable biomass

Calculate the effluent SMP

- let's first calculate the individual terms for Eqs. [3.38] & [3.39]

$$r_{ut} = -\frac{\hat{q}S}{K+S}X_a = \frac{dS}{dt} = -\frac{S^0 - S}{\theta} = -\frac{(500-1.7)\ mg\ BOD_L/L}{2\ d} = -249\ mg\ BOD_L/L-d$$

$$\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta = 1.8 \cdot 161 \cdot 2 + 100 + 0.12 \cdot (-249) \cdot 2 = 620\ mg\ BOD_L/L$$

$$4K_{UAP}k_1 r_{ut}\theta = 4 \cdot 100 \cdot 0.12 \cdot (-249) \cdot 2 = -23900\ (mg\ BOD_L/L)^2$$

$$K_{BAP}+(\hat{q}_{BAP}-k_2)X_a\theta = 85+(0.1-0.09)\cdot 161\cdot 2 = 88.2\ mg\ BOD_L/L$$

$$4K_{BAP}k_2X_a\theta = 4\cdot 85\cdot 0.09\cdot 161\cdot 2 = 9850\ (mg\ BOD_L/L)^2$$

$$UAP = \frac{-620+\sqrt{(620)^2+23900}}{2} = 9.5\ mg\ BOD_L/L$$

$$BAP = \frac{-88.2+\sqrt{(88.2)^2+9850}}{2} = 22.3\ mg\ BOD_L/L$$

$$SMP = UAP+BAP = 9.5+22.3 = 31.8\ mg\ BOD_L/L$$

Biomass COD: recall that the COD value for a cell formula of $C_5H_7O_2N$ was 1.42 g COD/g cells

In sum,

$$Effluent\ COD = {}_{strate} + SMP + Biomass\ COD$$

$$= 1.7+31.8+(1.42\ g\ COD/g\ VSS)X_v = 1.7+31.8+1.42\cdot 221$$

$$= 1.7+31.8+313.8 = 347\ mg\ COD/L$$

* *Biomass accounts for most of COD – this COD can be removed by settling (but good settling property should be guaranteed)*

* *SMP account for most of soluble COD*

$$Effluent\ BOD_L = {}_{strate} + SMP + active \text{ and } biodegradable\ biomass$$

$$= 1.7+31.8+(1.42\ g\ COD/g\ VSS)\cdot X_a\cdot f_d = 216\ mg\ BOD_L/L$$

3) N and P

The N and P consumption rates,

$$r_N = (0.124\ g\ N/g\ VSS)\cdot(0.42\ g\ VSS/g\ \ BOD_L)\cdot(-249\ mg\ BOD_L/L-d)\cdot\frac{1+(1-0.8)\cdot 0.15\cdot 2}{1+0.15\cdot 2}$$

$$= -10.6\ mg\ N/L-d$$

$$r_P = r_N\cdot 0.2\ g\,P/g\,N = -10.6\cdot 0.2 = -2.1\ mg\ \ P/L-d$$

The effluent N and P concentrations

$$C_N = C_N^0 + r_N\theta = 50\ mg\ N/L-(10.6\ mg\ N/L-d)\cdot 2\ d = 28.8\ mg\ NH_4^+-N/L$$

$$C_P = C_P^0 + r_P\theta = 10\ mg\ P/L-(2.1\ mg\ P/L-d)\cdot 2\ d = 5.8\ mg\ PO_4^{3-}-P/L$$

(the amount of nutrients did not limit the biological activity in the reactor)

4) O_2

The acceptor consumption in the reactor,

$$-\frac{\Delta S_a}{\Delta t} = (1\ g\ O_2/g\ COD)\cdot(1000\ m^3/d)$$

$$\cdot\ [500-1.7-31.8+1.42(50-221)]\,mg\ COD/L\cdot 10^3\ L/m^3\cdot 10^{-3}g/mg$$

$$= 2.24\times 10^5\ g\ O_2/d$$

To support the acceptor consumption, O_2 should be supplied to the reactor with a rate of:

$$R_{O_2} = 2.24\times 10^5\ g\ O_2/d - (1000\ m^3/d)\cdot(6-2)\ mg/L\cdot 10^3\ L/m^3\cdot 10^{-3}\ g/mg$$

$$= 2.20\times 10^5\ g\ O_2/d$$

(O_2 supplied by the influent DO is very small compared to the O_2 requirement – aeration is essential)

5) Effect of hydrolysis

i) effluent particulate BOD, S_p

$$S_p = \frac{S_p^0}{1+k_{hyd}\theta} = \frac{100\ mg\ COD/L}{1+(0.2/d)(2\ d)} = 71\ mg\ COD/L$$

ii) effluent soluble BOD, S: no change, 1.7 mg BODL/L

iii) effective S^0 considering S_p, $S^{0\prime}$:

$$S^{0\prime} = S^0 + k_{hyd}S_p\theta = 500\ mg\ COD/L + (0.2/d)(71\ mg\ COD/L)(2\ d) = 528\ mg\ BOD_L/L$$

(error in the textbook!)

iv) Effluent VSS

$$X_a = Y(S^0 - S)\frac{1}{1+b\theta_x} = 0.42\cdot(528-1.7)\frac{1}{1+0.15\cdot 2} = 170\ mg\ VSS/L$$

(slight increase from 161 mg VSS/L without particulate BOD)

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x = 50\ mg\ VSS/L + (170\ mg\ VSS/L)\cdot(1-0.8)\cdot(0.15/d)\cdot(2\ d)$$

$$= 60\ mg\ VSS/L$$

(didn't change much – slight increase happened, but not enough to have increase in significant numbers)

$$X_v = X_a + X_i + S_p = (170+60)\ mg\ VSS/L + \frac{71\ mg\ COD/L}{1.42\ mg\ COD/mg\ VSS} = 280\ mg\ VSS/L$$

(Assumed that the particulate COD has the same formula, $C_5H_7O_2N$, as the biomass)

v) SMP: let's skip the calculation and obtain value from the text:
SMP = 32.6 mg BODL/L
(slight increase from 31.8 mg BOD_L*/L because of increased biomass - BAP increases)*
vi) Effluent COD & BOD_L

$$Effluent\ COD = S + SMP + 1.42 \cdot X_v = 1.7 + 32.6 + 1.42 \cdot 280 = 432\ mg\ COD/L$$

$$Effluent\ BOD_L = S + SMP + 1.42 \cdot f_d \cdot X_a + S_p = 1.7 + 32.7 + 1.42 \cdot 0.8 \cdot 170 + 71 =$$
$$= 299\ mg\ BOD_L/L$$