Homework #4 - Solutions

Due: June 02 (Tue) 23:59

1. Derive the overall mass transfer coefficient with gas phase as a reference, K_G, for air-water interface as a function of k_L, k_G, and H_{cc}. Recall that the overall mass transfer coefficient with liquid phase as a reference is given as

$$K_L = \frac{k_L k_G H_{cc}}{k_L + k_G H_{cc}}$$ (L32 lecture note slide 8)

Derive the relationship between K_L and K_G.
(30 points)

Answer)

$$J_{tot} = K_G\left(C^G_{bulk} - C^G_{bulk}*\right) = K_G\left(C^G_{bulk} - H_{cc}C^L_{bulk}\right)$$

(set gas → liquid as positive flux)

$$J_{tot} = k_L\left(C^L_{int} - C^L_{bulk}\right)$$

$$C^L_{int} - C^L_{bulk} = \frac{J_{tot}}{k_L}$$

$$J_{tot} = k_G\left(C^G_{bulk} - C^G_{int}\right)$$

$$C^G_{bulk} - C^G_{int} = \frac{J_{tot}}{k_G}$$

$$J_{tot} = K_G\left\{C^G_{bulk} - C^G_{int} + H_{cc}\left(C^L_{int} - C^L_{bulk}\right)\right\}$$

$$= K_G\left(\frac{J_{tot}}{k_G} + H_{cc}\frac{J_{tot}}{k_L}\right)$$

$$\frac{J_{tot}}{K_G} = \frac{J_{tot}}{k_G} + \frac{J_{tot}}{k_L/H_{cc}}$$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{1}{k_L/H_{cc}} \quad or \quad \underline{K_G = \frac{k_L k_G}{k_L + k_G H_{cc}}}$$

Relationship between K_L and K_G:

$K_L = H_{cc} \times K_G$

2. The K_La value for oxygen in an aerobic bioreactor is determined to be 5.0 hr^{-1}. What would be the K_La value for nitrous oxide (N_2O), a greenhouse gas that may be produced by biological reactions, in the bioreactor? Use the following data and assumptions.

* H_{cc} (oxygen) = 30; H_{cc} (nitrous oxide) = 1.7
* Surface renewal theory applies to the gas-liquid mass transfer.
* $k_G/k_L \fallingdotseq 100$.
* Less than 5% error is negligible.
* Diffusion coefficient in water, D_{aq}
  D_{aq} (oxygen) = 2.0×10^{-9} m^2/s; D_{aq} (nitrous oxide) = 1.6×10^{-9} m^2/s

(20 points)

Answer)

As $R_G = \frac{1}{k_G H_{cc}} < \frac{1}{100}\frac{1}{k_L} = \frac{1}{100}R_L$ *for both oxygen and nitrous oxide,*

Liquid phase boundary layer controls the gas-liquid mass transfer for both compounds

If surface renewal theory holds, $k_L \propto D_{aq}^{0.5}$.

So: $K_L \propto D_{aq}^{0.5}$ *and so does* K_La *because* a *is a constant for the same bioreactor.*

K_La *(nitrous oxide)* = $5.0\,hr^{-1} \times \left(\frac{1.6\times10^{-9}\ m^2/s}{2.0\times10^{-9}\ m^2/s}\right)^{0.5} = 4.5\,hr^{-1}$