Analog Electronic Circuits
Department of Electrical and Computer Engineering
Seoul National University

2020 Fall D.K. Jeong

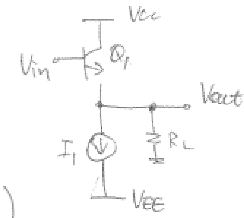
Homework Assignments #6

- Due: 2020/12/23
- There will be a 15-min. quiz on the homework questions before lecture.

1. Solve textbook problems in Chapter 14: 2, 19, 21, 25, and 37.

14.2

(a) $I_1 = V_P / R_L \qquad V_P \gg V_T$



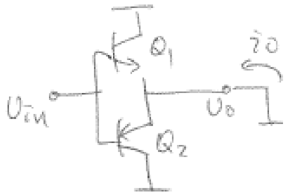
$$A_V = \frac{I_C R_L}{I_C R_L + V_T} = \frac{\frac{I_C}{I_1} V_P}{\frac{I_C}{I_1} V_P + V_T} = \frac{V_P}{V_P + V_T} \quad (\approx 1)$$

(b) When $V_{out} = V_P$, $I_{C_1} = I_1 + \frac{V_{out}}{R_L} = \frac{V_P}{R_L} + \frac{V_P}{R_L} = \frac{2V_P}{R_L}$

$$\therefore A_V = \frac{\left(\frac{2V_P}{R_L}\right) R_L}{\left(\frac{2V_P}{R_L}\right) R_L + V_T} = \frac{2V_P}{2V_P + V_T} \quad \left(\approx \frac{2V_P}{2V_P} = 1.\right)$$

$$\Delta A_V = \frac{\frac{2V_P}{2V_P + V_T} - \frac{V_P}{V_P + V_T}}{\frac{V_P}{V_P + V_T}} = \frac{V_T}{2V_P + V_T} \quad \left(\approx \frac{V_T}{V_P}\right)$$

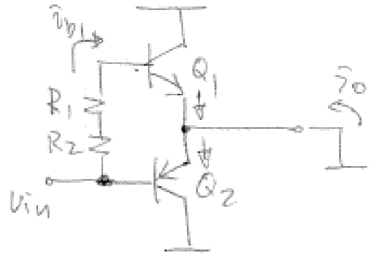
14.19



$$G_M = \frac{i_o}{v_{in}} = -(g_{m1} + g_{m2})$$

$$\Rightarrow \frac{v_o}{v_{in}} = \frac{i_o R_L}{v_{in}} = +(g_{m1} + g_{m2}) R_L$$

14.21



$$i_o = -g_{m1} v_{be1} + g_{m2}|v_{be2}| \quad (i_o = i_{c2} - i_{c1})$$

$$|v_{be2}| = v_{in}$$

$$v_{be1} = v_{in} - i_{b1}(R_1+R_2) = v_{in} - \frac{i_{c1}}{\beta_1}(R_1+R_2)$$

$$= v_{in} - \frac{i_{c2} - i_o}{\beta_1}(R_1+R_2)$$

$$= v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)$$

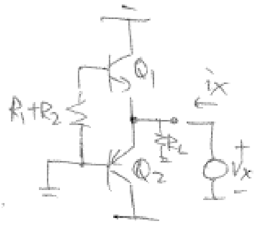
$$\therefore g_{m1}\left[v_{in} + \frac{g_{m2} v_{in} + i_o}{\beta_1}(R_1+R_2)\right] + i_o = -g_{m2} v_{in}$$

Solving for $\frac{i_o}{v_{in}}$ gives:

$$G_m = \frac{i_o}{v_{in}} = \frac{-\left[g_{m1} + \frac{g_{m1} g_{m2}}{\beta_1}(R_1+R_2) + g_{m2}\right]}{1 + \frac{g_{m1}(R_1+R_2)}{\beta_1}}$$

Rout:

$$\frac{v_x}{i_x} = R_{out} = \left(r_{\pi 2} \| \frac{1}{g_{m2}}\right) \| \left[(r_{\pi 1} + R_1 + R_2) \| \frac{1}{g_{m1}}\right] \| R_L$$



$$\therefore A_v = G_m R_{out}$$

$$= -\left[\frac{g_{m1} + \frac{g_{m1} g_{m2}(R_1+R_2)}{\beta_1} + g_{m2}}{1 + \frac{g_{m1}(R_1+R_2)}{\beta_1}}\right] \cdot \left\{\left[r_{\pi 2} \| \frac{1}{g_{m2}}\right] \| \left[(r_{\pi 1} + R_1 + R_2) \| \frac{1}{g_{m1}}\right] \| R_L\right\}$$

14.25

$$\frac{V_x}{I_x} = \frac{1}{g_{m1}+g_{m2}} + \frac{r_{O3} \parallel r_{O4}}{(g_{m1}+g_{m2})(r_{\pi 1} \parallel r_{\pi 2})}$$

If $g_{m1} \approx g_{m2} = g_m$:

$$\frac{V_x}{I_x} \approx \frac{1}{2g_m} + \frac{r_{O3} \parallel r_{O4}}{2g_m\left(\frac{\beta_1}{g_m} \parallel \frac{\beta_2}{g_m}\right)}$$

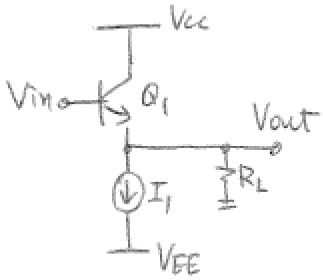
$$= \frac{1}{2g_m} + \frac{r_{O3} \parallel r_{O4}}{2g_m\left(\frac{1}{g_m}\,\frac{\beta_1\beta_2}{\beta_1+\beta_2}\right)}$$

$$= \frac{1}{2g_m} + \frac{r_{O3} \parallel r_{O4}}{2\beta_1\beta_2}(\beta_1+\beta_2)$$

14.37

Power = 1 W

$R_L = 8\Omega$



$$P_{LOAD} = \frac{1}{2}\frac{V_p^2}{R_L} = 1W$$

$\Rightarrow V_p = 4\,V \Rightarrow I_1 = \frac{V_p}{R_L} = 0.5\text{ mA}$

(Note: the problem does not specify small-signal voltage gain, so choose $V_p = I_1 R_L$)

P_{Q_1} (power rating) $= I_1(V_{cc})$

$= (0.5mA)(5V)$

$= 2.5\text{ mW}$

2. Consider the following circuit shown in Fig. 1. Assume that $V_{CC} = 15V$, $V_{EE} = -15V$, $V_T = 26mV$, $I_{S,Q1} = I_{S,Q4} = 10^{-14}A$, $I_{S,Q2} = I_{S,Q3} = 10^{-13}A$, and $V_A = \infty$. Also assume that the output is sinusoidal with maximum amplitude of 12V (i.e. $V_{OUT} = 12\sin\omega t$).

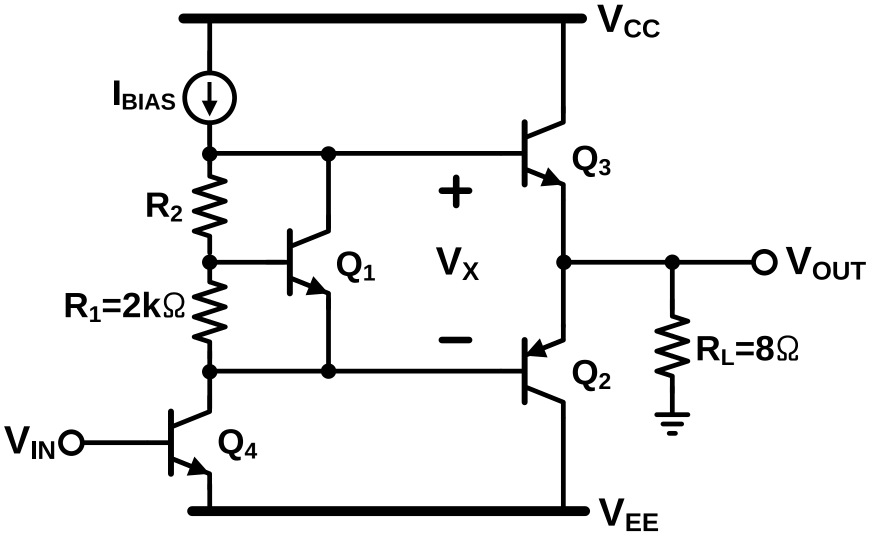
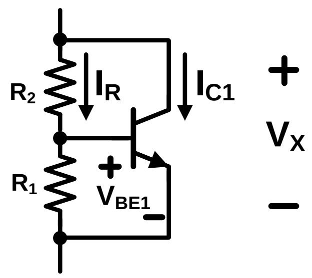


Fig. 1

(a) Calculate V_X using R_1, R_2, and V_{BE1}. Assume that $\beta_{Q1} = \infty$.



$V_X = (R_1 + R_2)I_R$ - (1)

$V_{BE1} = R_1 I_R$ - (2)

➔ $V_X = \left(1 + \frac{R_2}{R_1}\right) V_{BE1}$

(b) Calculate the minimum value of the bias current, I_{BIAS}. Assume that $\beta_{Q2}=\beta_{Q3}=100$.

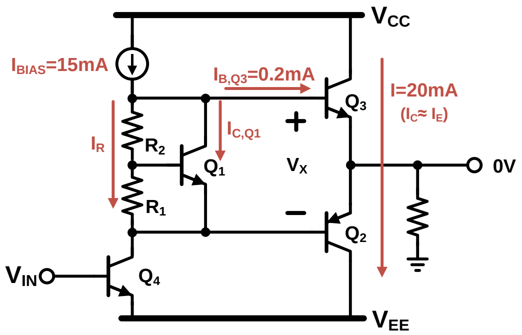
$V_{OUT}=12V$ 일 때 I_{B3} 가 최대이므로, 이 때 $I_{B,Q3} \le I_{BIAS}$ 조건을 만족시켜야 함.

➔ $I_{C,Q3} = \frac{12V}{8\Omega} = (\beta+1)I_{B,Q3}$

➔ $I_{B,Q3} \approx 15mA$

$\therefore I_{BIAS} \ \ge \ 15mA \ \rightarrow I_{BIAS,min} \ = \ 15mA$

(c) Using the value of (b), find the value of R_2 to set the quiescent current at the output transistors to 20mA. (Quiescent current condition : V_{OUT}=0)



$V_X = V_{BE3} + V_{EB2} = V_T ln\frac{I_{C,Q3}}{I_{S,Q3}} + V_T ln\frac{I_{C,Q2}}{I_{S,Q2}} = 2 \times 26\text{mV} \times \ln\frac{20mA}{10^{-13}A} = 1.353V$ – (3)

$V_{BE1} = V_T ln\frac{I_{C,Q1}}{I_{S,Q1}} = 26\text{mV} \times \ln\frac{15mA-200uA-I_R}{10^{-14}A} = 2k\Omega \times I_R \ \rightarrow I_R \ \approx 364uA$ – (4)

(3), (4)에 의해 $V_X = (R_1 + R_2)I_R \rightarrow 1.353V = (2000 + R_2)364uA$

$\therefore R_2 \approx 1.72k\Omega$

(d) What is the power efficiency? Ignore the power dissipation at the pre-driver.

$\eta = \frac{P_{OUT}}{P_{OUT}+P_{av,Q2}+P_{av,Q3}} = \frac{\pi}{4}\frac{V_P}{V_{CC}} = \frac{\pi}{4}\frac{12}{15} = 0.628$

$\therefore Efficiency = 62.8\%$

<End of HW #6>