Example question: Interphase mass transfer

1. Derive the overall mass transfer coefficient with gas phase as a reference, K_G, for air-water interface as a function of k_L, k_G, and H_{cc}. Recall that the overall mass transfer coefficient with liquid phase as a reference is given as

$$K_L = \frac{k_L k_G H_{cc}}{k_L + k_G H_{cc}} \qquad \text{(lecture note slide 20)}$$

Derive the relationship between K_L and K_G.
(30 points)

Answer)

$J_{tot} = K_G\left(C^G_{bulk} - H_{cc}C^L_{bulk}\right)$

(set gas → liquid as positive flux)

$J_{tot} = k_L\left(C^L_{int} - C^L_{bulk}\right)$

$C^L_{int} - C^L_{bulk} = \frac{J_{tot}}{k_L}$

$J_{tot} = k_G\left(C^G_{bulk} - C^G_{int}\right)$

$C^G_{bulk} - C^G_{int} = \frac{J_{tot}}{k_G}$

$J_{tot} = K_G\left\{C^G_{bulk} - C^G_{int} + H_{cc}\left(C^L_{int} - C^L_{bulk}\right)\right\}$

$= K_G\left(\frac{J_{tot}}{k_G} + H_{cc} \cdot \frac{J_{tot}}{k_L}\right)$

$\frac{J_{tot}}{K_G} = \frac{J_{tot}}{k_G} + \frac{J_{tot}}{k_L/H_{cc}}$

$\frac{1}{K_G} = \frac{1}{k_G} + \frac{1}{k_L/H_{cc}}$ *or* $\underline{K_G = \frac{k_L k_G}{k_L + k_G H_{cc}}}$

Relationship between K_L and K_G:

$K_L = H_{cc} \times K_G$

2. You set up an experiment to determine the overall mass transfer

coefficient, K_{tot}, for gas-liquid transfer of oxygen and ethanol in a well-mixed reactor. You also need to estimate the K_{tot} for nitrogen, which was not determined by the experiment. The results of the experiment, the unitless Henry's law constants, and the diffusion coefficients in water are as follows:

Compounds	K_{tot} (m/s)	H_{cc}	D_w (m^2/s)
Nitrogen, N_2	-	65	1.9×10^{-9}
Oxygen, O_2	2.0×10^{-5}	30	2.1×10^{-9}
Ethanol, C_2H_5OH	4.1×10^{-7}	2.7×10^{-4}	1.2×10^{-9}

i) Determine whether gas or liquid phase will control the mass transfer for each compound assuming a k_G/k_L of 100. Show your reasoning. (20 points)

Answer)

For nitrogen: $\frac{R_L}{R_G}=\frac{k_G H_{CC}}{k_L}=100\times65=6500\gg1$; *liquid phase controls*

For oxygen: $\frac{R_L}{R_G}=\frac{k_G H_{CC}}{k_L}=100\times30=3000\gg1$; *liquid phase controls*

For ethanol: $\frac{R_L}{R_G}=\frac{k_G H_{CC}}{k_L}=100\times2.7\times10^{-4}=0.027\ll1$; *gas phase controls*

ii) Assuming that film theory is applicable, predict the K_{tot} for nitrogen from the K_{tot} data for oxygen. (20 points)

Answer)

According to film theory, $k_L\propto D_L$

For both nitrogen and oxygen, liquid phase controls the mass transfer, so $K_{tot}\approx k_L$

$$\frac{k_{L,N_2}}{k_{L,O_2}}=\frac{D_{L,N_2}}{D_{L,O_2}}$$

$$K_{tot,N_2}=k_{L,N_2}=k_{L,O_2}\times\frac{D_{L,N_2}}{D_{L,O_2}}=(2.0\times10^{-5}\ m/s)\times\frac{1.9\times10^{-9}\ m^2/s}{2.1\times10^{-9}\ m^2/s}=1.8\times10^{-5}\ m/s$$

3. You sampled 200 cm^3 of water with a dissolved oxygen (DO) concentration of 5.0 mg/L in a beaker with a diameter of 5 cm. How long will it take for the DO to reach 6.5 mg/L? Use the overall mass

transfer coefficient, K_L, of 0.01 m/h for still water in indoor environment suggested in the literature. Use the saturation DO concentration of 9.2 mg/L at 20℃. (30 points)

Answer)

$$a = \frac{A}{V} = \frac{(5\ cm)^2 \cdot \pi/4}{200\ cm^3} = 0.098/cm$$

$$K_L a = 0.01\ m/h \times 0.098/cm \times 10^2\ cm/m = 0.098/h$$

$$\frac{dC_{bulk}^{L}}{dt} = -K_L a\left(C_{bulk}^{L} - C_{bulk}^{L*}\right)$$

$$\frac{dC_{bulk}^{L}}{C_{bulk}^{L} - C_{bulk}^{L*}} = -K_L a$$

$$\int_{C_{bulk}^{L}(t=0)}^{C_{bulk}^{L}(t=t)} \frac{dC_{bulk}^{L}}{C_{bulk}^{L} - C_{bulk}^{L*}} = -K_L a \int_0^t dt$$

$$\ln \frac{C_{bulk}^{L}(t=t) - C_{bulk}^{L*}}{C_{bulk}^{L}(t=0) - C_{bulk}^{L*}} = -K_L a \cdot t$$

$$t = -\frac{1}{K_L a} ln \frac{C_{bulk}^{L}(t=t) - C_{bulk}^{L*}}{C_{bulk}^{L}(t=0) - C_{bulk}^{L*}} = -\frac{1}{0.098/h} ln \frac{6.5 - 9.2}{5.0 - 9.2} = 4.5\ h$$