Fusion Plasma Theory I

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Homework 1

Due: March 29, 2021

1. [**Cylindrical Debye Shielding**] Suppose a very long and thin wire with a total charge Q inserted into a hydrogen unmagnetized plasma which otherwise would remain uniformly neutral with the density n. Assume the electron and ion temperature $T_e = T_i$. Find the distribution of the electric potential ϕ in space due to the wire and verify that it becomes exponentially small after a few Debye lengths.

2. [**Sheath Thickness**] The ion saturation current I_{is} from a material probe inserted in a plasma actually depends slightly on the applied voltage V (or ϕ_0), i.e., increasing rather than saturated when the voltage becomes more and more negative. Why? Assuming cold ions deep in unmagnetized plasma, and taking the plasma potential as a reference point i.e. $\phi = 0$, find the dependency of the ion saturation current due to the applied voltage $I_{is}(V)$ quantitatively. Assume a cylindrical probe with the radius a.

3. [**Charge Separation (GR: Problem 3.4)**] An anisotropic proton-electron plasma is immersed in the magnetic field from an infinite wire carrying current $I_z = 1MA$. This plasma has uniform density $n = 10^{19}\text{m}^{-3}$, $T_{\perp e} = T_{\perp i} = 2\text{keV}$ and $T_{\|,e} = T_{\|,i} = 5\text{keV}$.

(a) At radius R away from the wire, what are the average ion and electron $\vec{\nabla} B$ and curvature drift velocities?

(b) What is the total guiding center current density, $\vec{j} = \sum nq\vec{v}$, in the plasma, and in which direction does this current flow? (Ignore the magnetic field due to the current in the plasma.)

4. [Magnetic Pumping (GR: Problem 4.2)] Imagine that you have an isotropic magnetized plasma with with $T_{\|0} = T_{\perp 0} = T_0$.

(a) Double the magnetic field slowly compared to a gyro-period, but fast compared to the energy transfer time between $T_\|$ and $T_\perp$. What are the new values of $T_\|$ and $T_\perp$ (call them $T_{\|1}$ and $T_{\perp 1}$)?

(b) Now let the plasma sit long enough for $T_{\|1}$ and $T_{\perp 1}$ to mix by collisions and come to an isotropic temperature T_1, but not long enough for the plasma to exchange energy with the outside world. What is T_1?

(c) Reduce the magnetic field back down to its original value slowly compared to a gyro-period, but fast compared to the energy transfer time between $T_\|$ and $T_\perp$. What are $T_{\|2}$ and $T_{\perp 2}$?

(d) Lastly, what is T_2 after the plasma becomes isotropic?

5. [Asymptotics, Optional] Have you read the Martin Kruskal's article, in particular the parts of the simple algebraic example? Can you then find the leading behavior of all of the roots to the equation, with a small ordering parameter $\epsilon \to +0$?

$$\epsilon^2 x^8 - \epsilon x^6 + x - 2 = 0 \tag{1}$$