

HW2.

The governing equation is as follows:

$$\frac{\partial \phi}{\partial t} - \alpha \frac{\partial^2 \phi}{\partial x^2} = 0, \quad 0.1 \leq x \leq 1.5.$$

Boundary conditions & initial condition are:

$$\frac{\partial \phi}{\partial x} = c = 2 - \pi \sin\left(\frac{\pi}{30}x\right) e^{-\alpha\left(\frac{\pi}{3}\right)^2 t} \quad @ \quad x = 0.1$$

$$\phi = 3 \quad @ \quad x = 1.5$$

$$\phi = 2x + 3 \cos\left(\frac{\pi}{3}x\right) e^{-\alpha\left(\frac{\pi}{3}\right)^2 t} \quad @ \quad t = 0.9$$

Exact solution of this problem is as follows:

$$\phi_{exact} = 2x + 3 \cos\left(\frac{\pi}{3}x\right) e^{-\alpha\left(\frac{\pi}{3}\right)^2 t}.$$

$$s = \frac{\alpha \Delta t}{\Delta x^2} = 0.2, \quad \alpha = 0.25$$

Compute corresponding root-mean-square (rms) errors

t=10.7	$\Delta x = 0.35$	$\Delta x = 0.175$	$\Delta x = 0.0875$	Convergence rate r
②'+③'				
②'+⑤'				

Compute error distribution with $\Delta x=0.35$ at $t=10.7$

t=10.7	$x = 0.1$	0.45	0.8	1.15	1.5
②'+③'					
②'+⑤'					