HW3.

Governing equation:

$$\frac{\partial}{\partial x}(\rho u\phi)=\frac{\partial}{\partial x}(\Gamma\frac{\partial\phi}{\partial x}),$$

Exact solution:

$$\phi=\phi_0+\frac{e^{xPe/L}-1}{e^{pe}-1}(\phi_L-\phi_0),$$

where $Pe=\rho uL/\Gamma, Pe_\Delta=\rho u\Delta/\Gamma$.

1. # of grid pts, N=11 including boundary points (uniform grids).
   - 1-1. Convection term for UDS
   - 1-2. Convection term for CDS
2. # of grid pts, N=41 including boundary points (uniform grids).
   - 1-1. Convection term for UDS
   - 1-2. Convection term for CDS
3. Non-uniform grids with N=11 ($\Delta x_i = re\Delta x_{i-1}, r_e = 0.75$)
   - 1-1. Convection term for UDS
   - 1-2. Convection term for CDS

The condition for the numerical simulations:

$$L=1,\rho=1,u=1,\Gamma=0.02$$

$$\phi_0=0,\phi_L=1,Pe=\frac{\rho uL}{\Gamma}=50$$

Plot the $\log\Delta x$ vs. $\log\varepsilon$ graph for $\varepsilon=Max\left|\phi_i^{exact}-\phi_i\right|$ and discuss the results.