H6_Derive lamellar spacing (λ) using Zener and Hillert equation

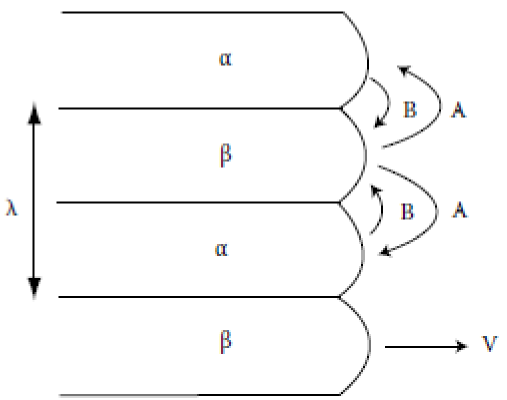

FIGURE 4.31
Interdiffusion in the liquid ahead of a eutectic front.

- **Inter-lamellar spacing λ**

For an inter-lamellar spacing, λ, there is a total of 2/λ of α and β interfaces. And the energy the gibb's fress energy change associated with the solidification for an inter-lamerllar spacing, λ, is

$$\Delta \mathrm{G}(\boldsymbol{\lambda}) = -\Delta \mathrm{G}(\infty) + \frac{2\gamma_{\alpha\beta} V_m}{\lambda}$$

Where ΔG(∞) is the free energy decrease for very large value of λ. And ΔG(∞) is given approximately by

$$\Delta \mathrm{G}(\infty) = \Delta H \cdot \frac{\Delta T_0}{T_E}$$

So the minimum possible spacing λ^* is

$$\lambda^* = \frac{2\gamma_{\alpha\beta} V_m T_E}{\Delta H \cdot \Delta T_E}$$

- **Eutectic growth rate v**

The velocity of eutectic growth is proportional to diffusivity and composition differences. And diffusion is inversely proportional to λ. So the velocity can be written by

$$v = k_1 D \frac{\Delta X}{\lambda}$$

ΔX will itself depend on λ for when $\lambda = \lambda^*$, ΔX=0, and the value is getting larger as λ is increased. Moreover the initial value of ΔX is proportional to ΔT_O. So ΔX and v can be written by

$$\Delta X = \Delta X_0 (1 - \frac{\lambda^*}{\lambda})$$

$$v = k_2 D \Delta T_0 \frac{1}{\lambda}(1 - \frac{\lambda^*}{\lambda})$$

With this relationship, we can know v having maximum value at $\lambda = 2\lambda^*$.

In conclusion, λ with maximum growth rate at a fixed ΔT is

$$\lambda^* = \frac{4\sigma_{\alpha/Fe_3C} T_E}{\Delta H_V \cdot \Delta T}$$

σ_{α/Fe_3C}= Interfacial energy per unit area of α/Fe_3C boundary
T_E= The equilibrium temperature (Ae_1)
ΔH_V = The change in enthalpy per unit volume
ΔT = The undercooling below Ae_1