Seoul National University
Active Aeroelasticity and Rotorcraft Lab.

Advanced Theory of Helicopter
Term project #1

Finite Element Method (FEM) for structural analysis

Prof. SangJoon Shin

Term Project
2020. 5. 12 / 15:30~16:00
Due date: 2020. 06. 19. 18:00

- **Term Project**
- **Theory**
- **EDISON proejct**
- **Conclusion**

- Term Project

- Theory

- EDISON proejct

Term project

❖ Make a complete code to calculate modes for rotating blade

➢ **Use any computer languages**

- Your own codes must be included in your term project
- If any question or hints, e-mail to goody147@snu.ac.kr

➢ **Use "EDISON project" to verify your results**

- Download "input.dat"
- If you change the input values, give you additional points.
- CPU run time is also important (+)
- If you find any bug, e-mail to goody147@snu.ac.kr for debugging.

Term project

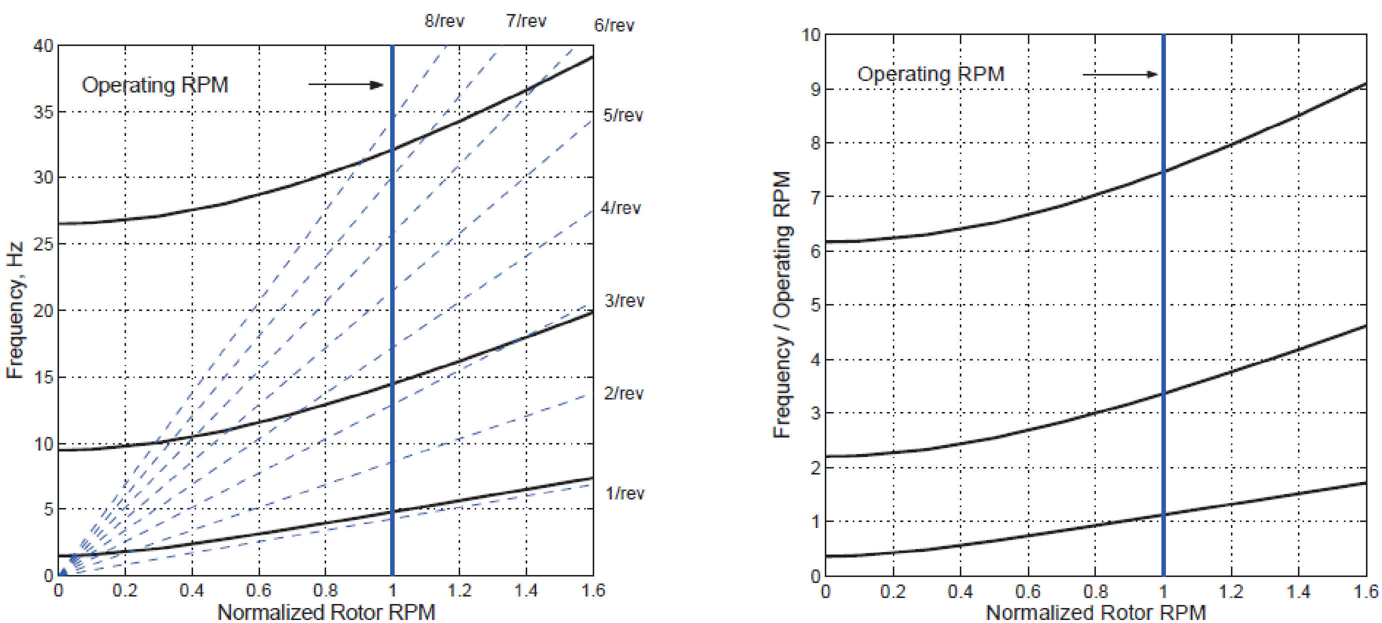
❖ Fan plot for rotating beams

➢ What is Fan plot for rotating beams?

- The natural modes of a structure represent the unique ways it can vibrate in vacuum and without damping

➢ The natural frequencies

- Depends on mass(M) and stiffness(K), boundary conditions(B.C), and rotational speed(Ω)



(a) Fan plot in Hz

(b) Frequency plot non-dimensionalized with respect to operating RPM

Term Project

❖ Calculate the non-dimensional Fan-plot

➢ Approach

- Utilize the Finite Element Method(FEM) for structural analysis
- Natural frequencies(f) ∝ ${}^{K}/_{M}$
- Two-dimensional beam

➢ Steps

- Discretize the rotating beam into a number of finite elements
  1. Develop the elemental properties
  2. Assemble the elemental properties
  3. Apply the constraints(B.C)

- Term Project
- **Theory**
- EDISON proejct

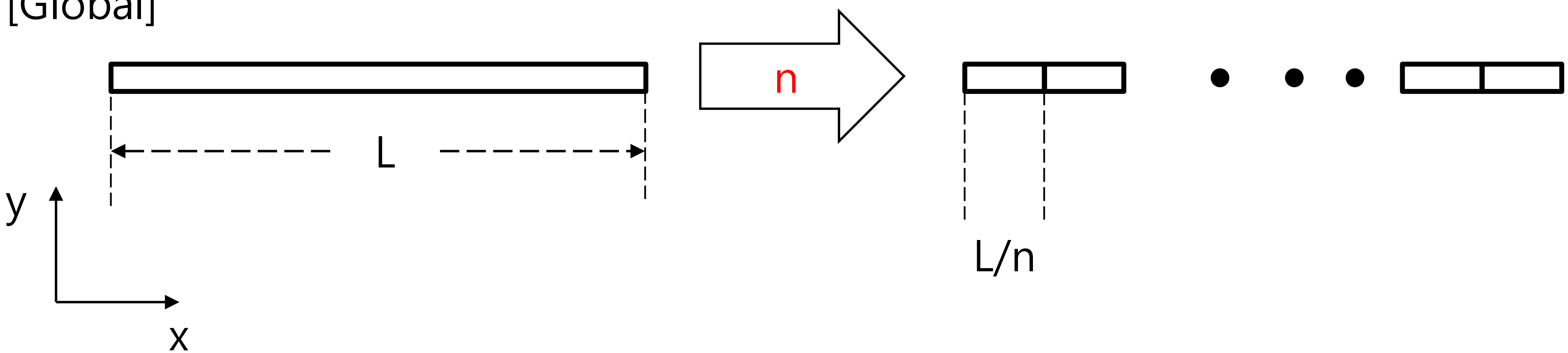
Theory

❖ Concepts

➢ Original geometry

- Discretize the rotating beam into a number of finite elements

[Global]



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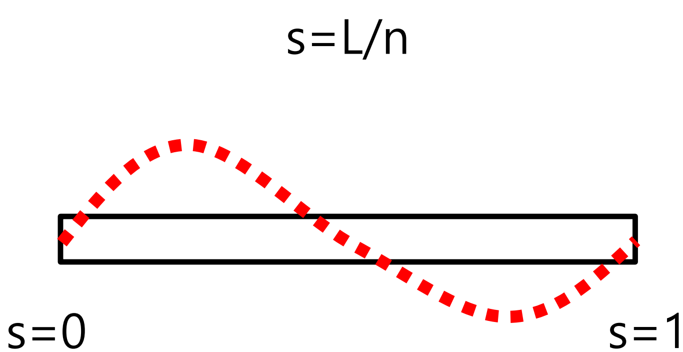


$$w(s) = \sum_{i=1}^{n} H_i(s) q_i(t)$$

$$H_i(s): interpolation\ function$$

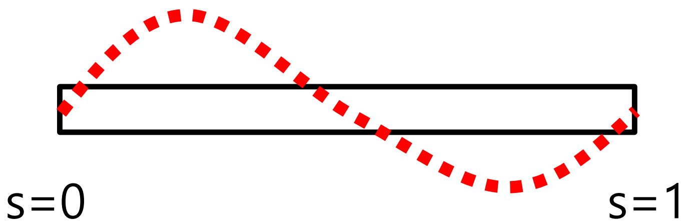
$$q_i(s): displacements$$

Theory

❖ Concepts

➢ **Assume interpolation function as a polynomial distribution w.r.t** w

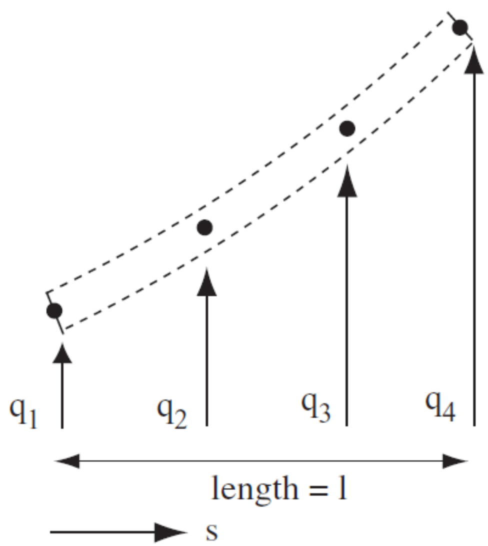
[Local]



$$w(s) = \sum_{i=1}^{n} H_i(s) q_i(t)$$

$H_i(s)$: *interpolation function*

$q_i(s)$: *displacements*



(c) Nodal displacements of the finite element

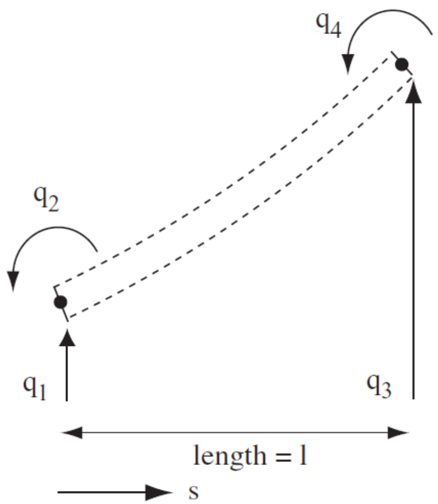
$$w(s,t) = \alpha_1 + s\alpha_2 + s^2\alpha_3 + s^3\alpha_4$$

$$w(0,t) = q_1 = \alpha_0$$
$$w(l/3,t) = q_2 = \alpha_0 + (l/3)\alpha_1$$
$$w(2l/3,t) = q_3 = \alpha_0 + (2l/3)\alpha_1 + (2l/3)^2\alpha_2 + (2l/3)^2\alpha_3$$
$$w(l,t) = q_4 = \alpha_0 + (l)\alpha_1 + (l)^2\alpha^2 + (l)^3\alpha_3$$

Theory

❖ Concepts

➢ **Solving for $\alpha_0 \sim \alpha_3$ in terms of $q_1 \sim q_4$**



(d) Nodal displacements and angles of the finite element

$$w(s,t) = \alpha_1 + s\alpha_2 + s^2\alpha_3 + s^3\alpha_4$$

$$w(0,t) = q_1 = \alpha_1$$
$$\frac{dw}{ds}(0,t) = q_2 = \alpha_2$$
$$w(l,t) = q_3 = \alpha_1 + \alpha_2(l) + \alpha_3(l)^2 + \alpha_4(l)^3$$
$$\frac{dw}{ds}(l,t) = q_4 = \alpha_2 + 2\alpha_3(l) + 3\alpha_3(l)^2$$

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & (l)^2 & (l)^3 \\ 0 & 1 & 2l & 3(l)^2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix}$$

Theory

❖ Concepts

➢ **General form of the interpolation functions**

$$w(s) = \sum_{i=1}^{n} H_i(s) q_i(t)$$

$$\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{Bmatrix} = \begin{bmatrix} H_1 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & H_3 & 0 \\ 0 & 0 & 0 & H_4 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}$$

$$H_1 = 2\left(\frac{s}{l}\right)^3 - 3\left(\frac{s}{l}\right)^2 + 1$$

$$H_2 = \left(\frac{s}{l}\right)^2 - 2\left(\frac{s}{l}\right) + s$$

$$H_3 = -2\left(\frac{s}{l}\right)^3 + 3\left(\frac{s}{l}\right)^2$$

$$H_4 = \left(\frac{s}{l}\right)^2 - \left(\frac{s}{l}\right)^2$$

Theory

❖ Concepts

➢ The elemental energies using the Rayleigh-Ritz method

- Total potential energy

  = the kinetic energy (U_e) + the strain energy (V_e) + ~~virtual work (Q_e)~~

- The kinetic energy (U_e)

$$U_e = \frac{1}{2}\int_0^l m\dot{w}^2\, ds = \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} m_{ij}\dot{q}_i\,\dot{q}_j$$

where, $m_{ij} = \rho\int_0^l H_i H_j\, ds$

- The strain enegy (V_e)

$$V_e = \frac{1}{2}\int_0^l EI(\ddot{w})^2 ds + \frac{1}{2}\int_0^R T(s)(\dot{w})^2 ds = \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} k_{ij} q_i q_j$$

where, $k_{ij} = EI\int_0^l \ddot{H}_i\,\ddot{H}_j ds + \int_0^l T(s)\dot{H}_i\,\dot{H}_j ds$

Theory

❖ Concepts

➢ The elemental energies using the Rayleigh-Ritz method

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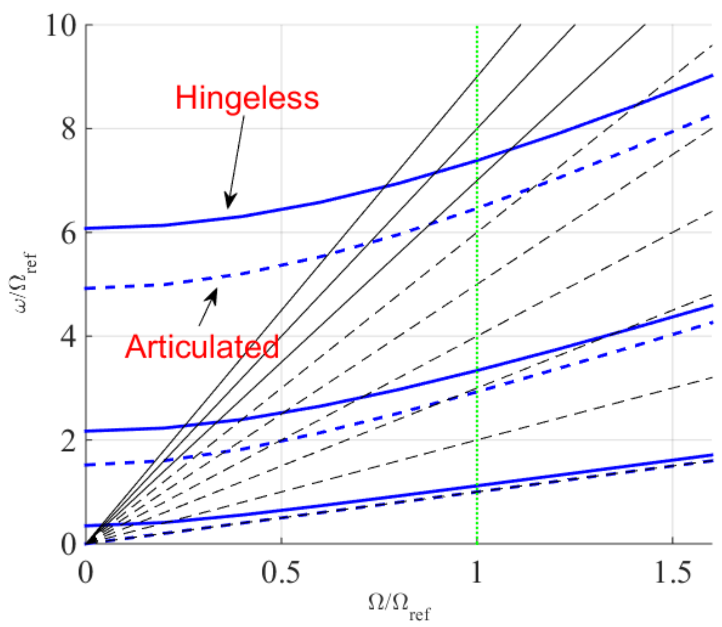
where, $k_{ij} = EI\int_0^l \ddot{H}_i\,\ddot{H}_j ds + \int_0^l T(s)\dot{H}_i\,\dot{H}_j ds$

Term project

❖ Rotating frequencies vs rotational speed (p.119)

- **Use following values**
  - R = 8.2 m
  - EI = 4.225×10^5
  - M = 13 kg/m
- **Fan-plot including both articulated and hingeless blade**
  - Plot first three modes w. r. t. non-dimensional rotating speed



- **Any computer language are allowed**
  - MATLAB, FORTRAN90, C …

Information

❖ Variables

➢ Geometrical variable

- Length (L) = 8.2 m
- Width (wid) = 0.4 m
- Height (h) = 0.04 m
- Area moment of inertia $I_{zz} = 1$

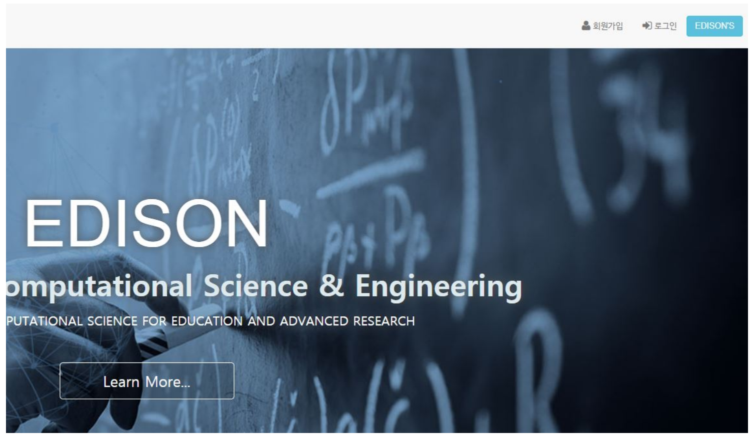
➢ Material variable

- Young's modulus (E) = 4.225×10^5
- Shear modulus (G) = 26.69×10^9
- Density (rho) = 13
- Nominal RPM (Non_omega) = 260

- Term Project
- Theory
- **EDISON proejct**

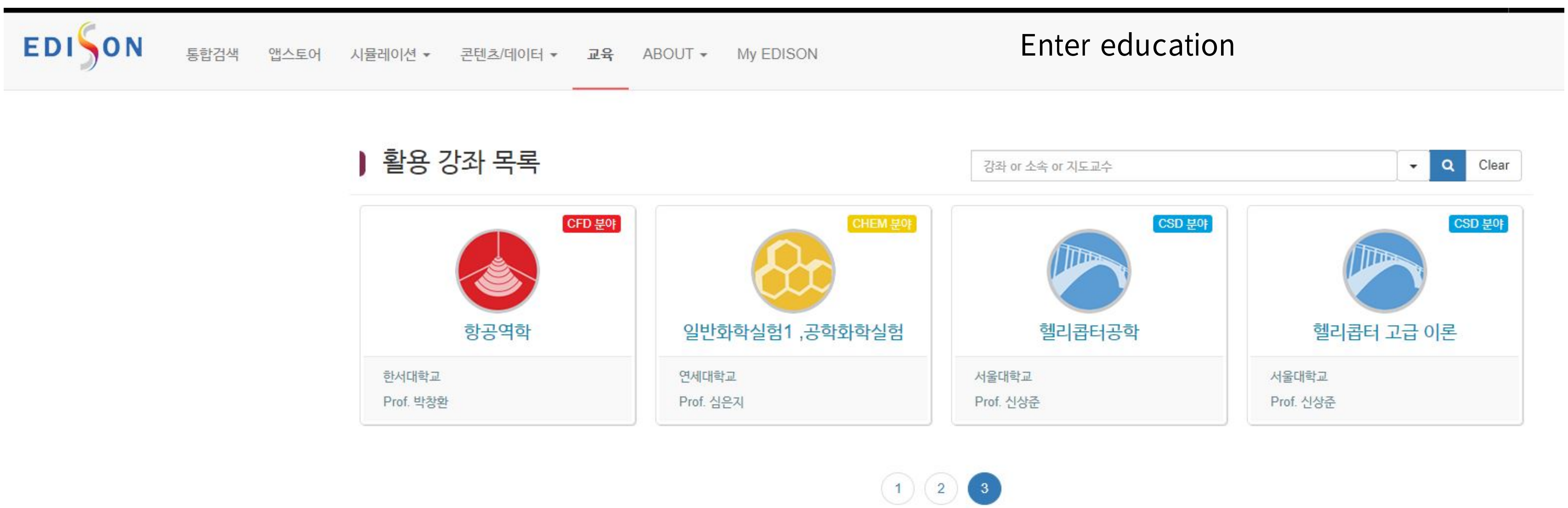
EDISON

❖ **csd.edison.re.kr**



Sign-up

Login



Enter education

enter Advanced helicopter

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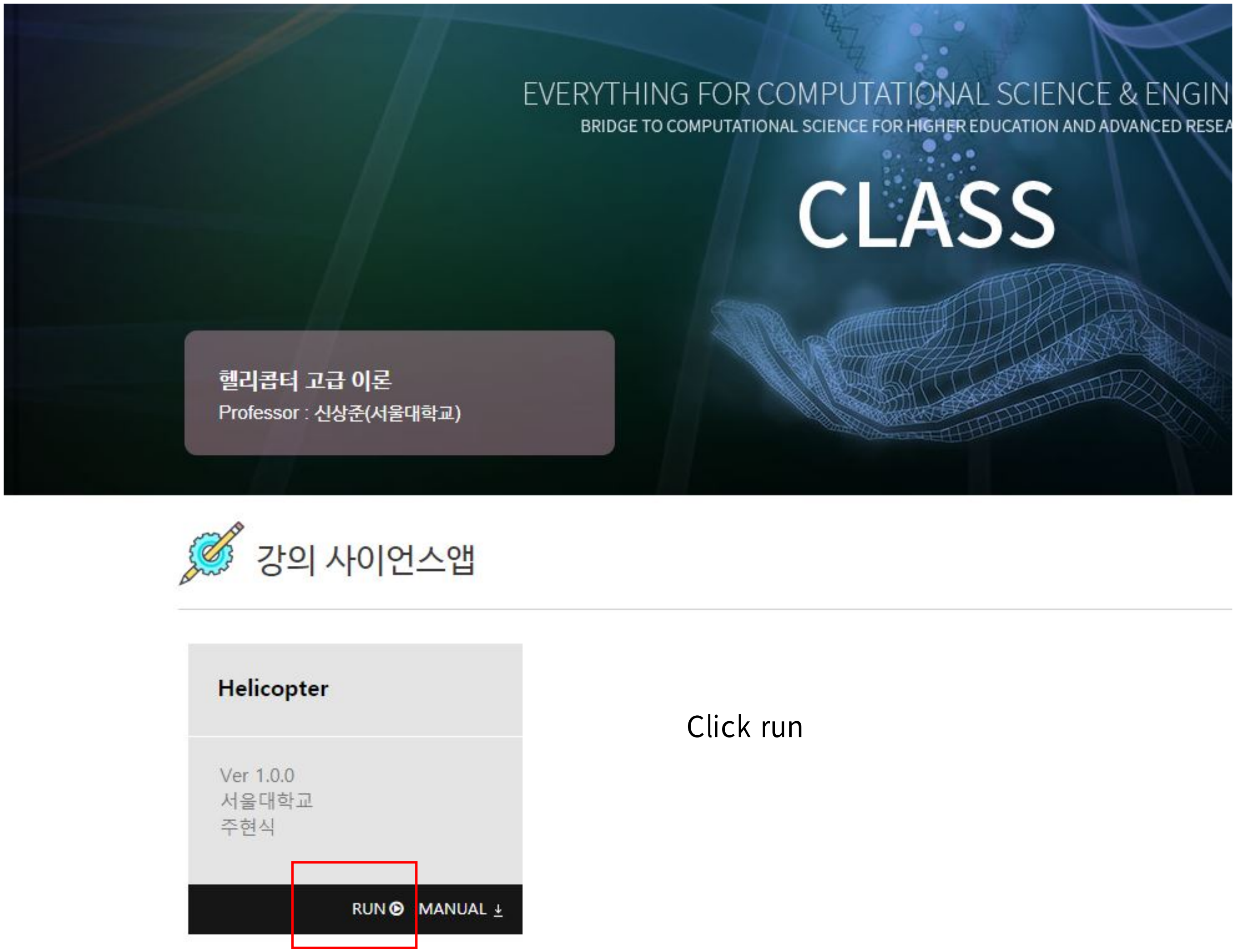

Click run

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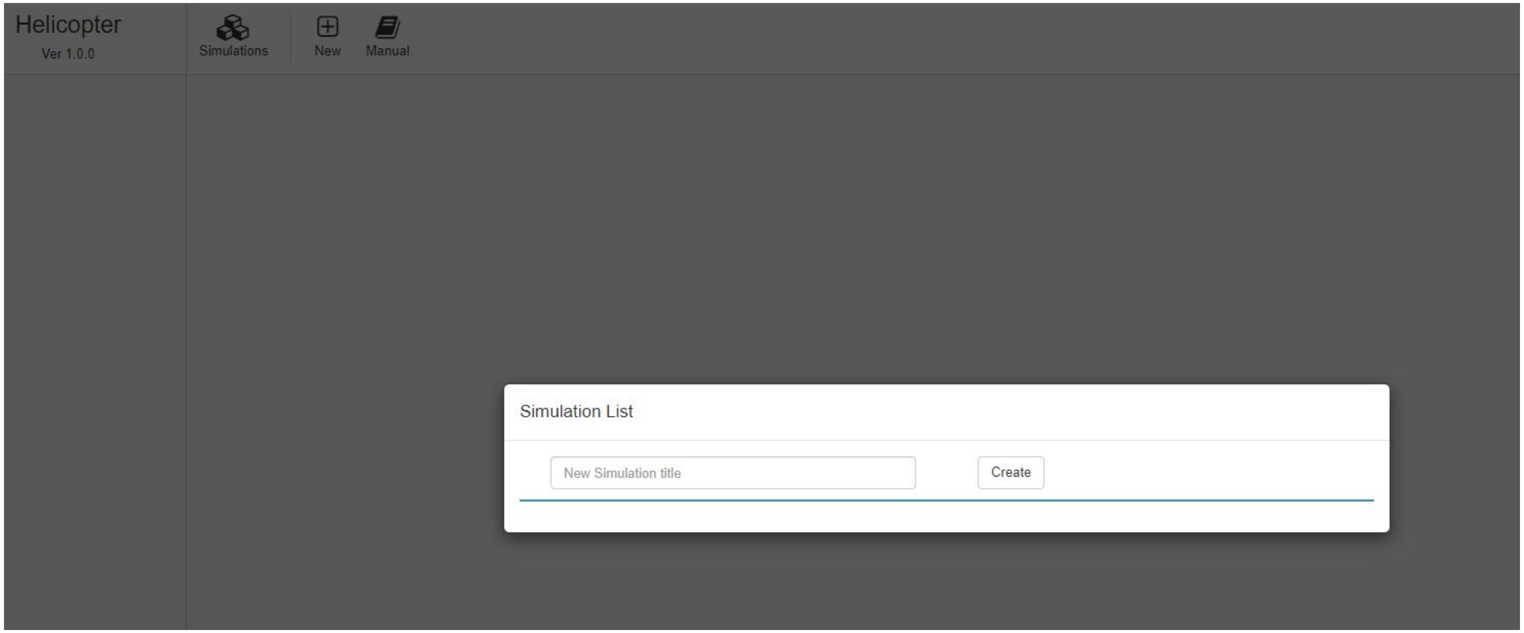

Enter simulation title

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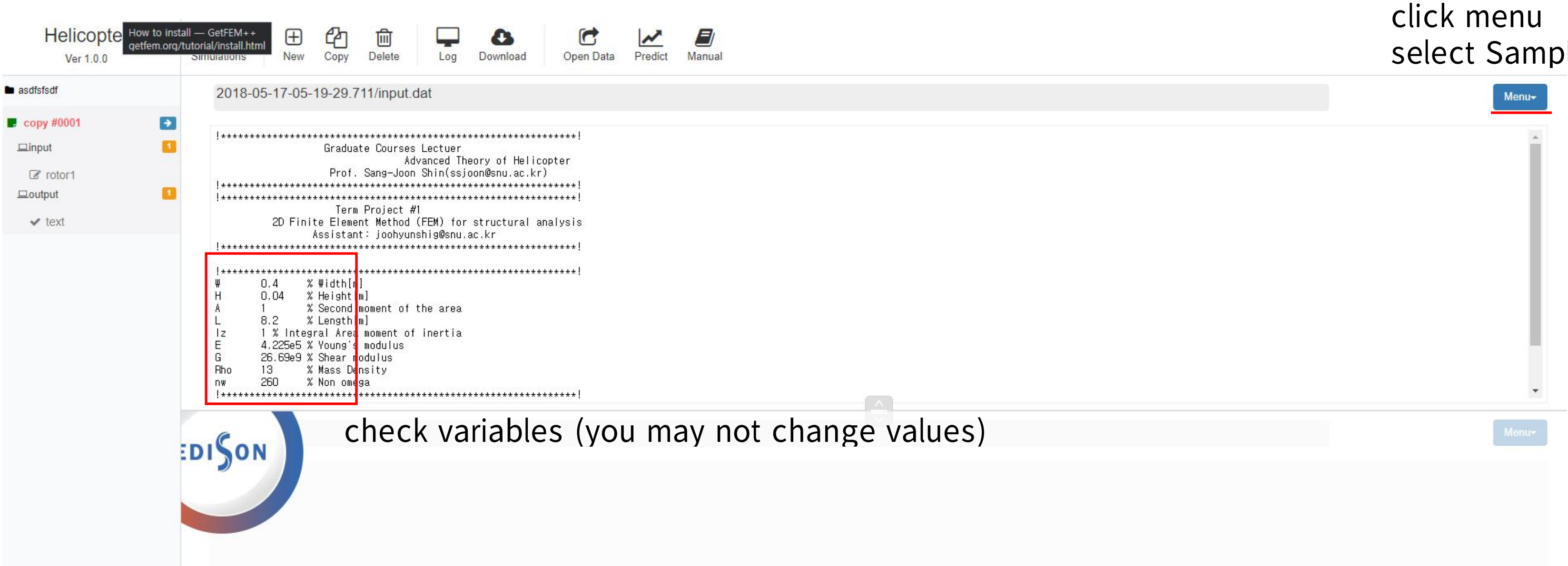
click menu
select Sample



check variables (you may not change values)

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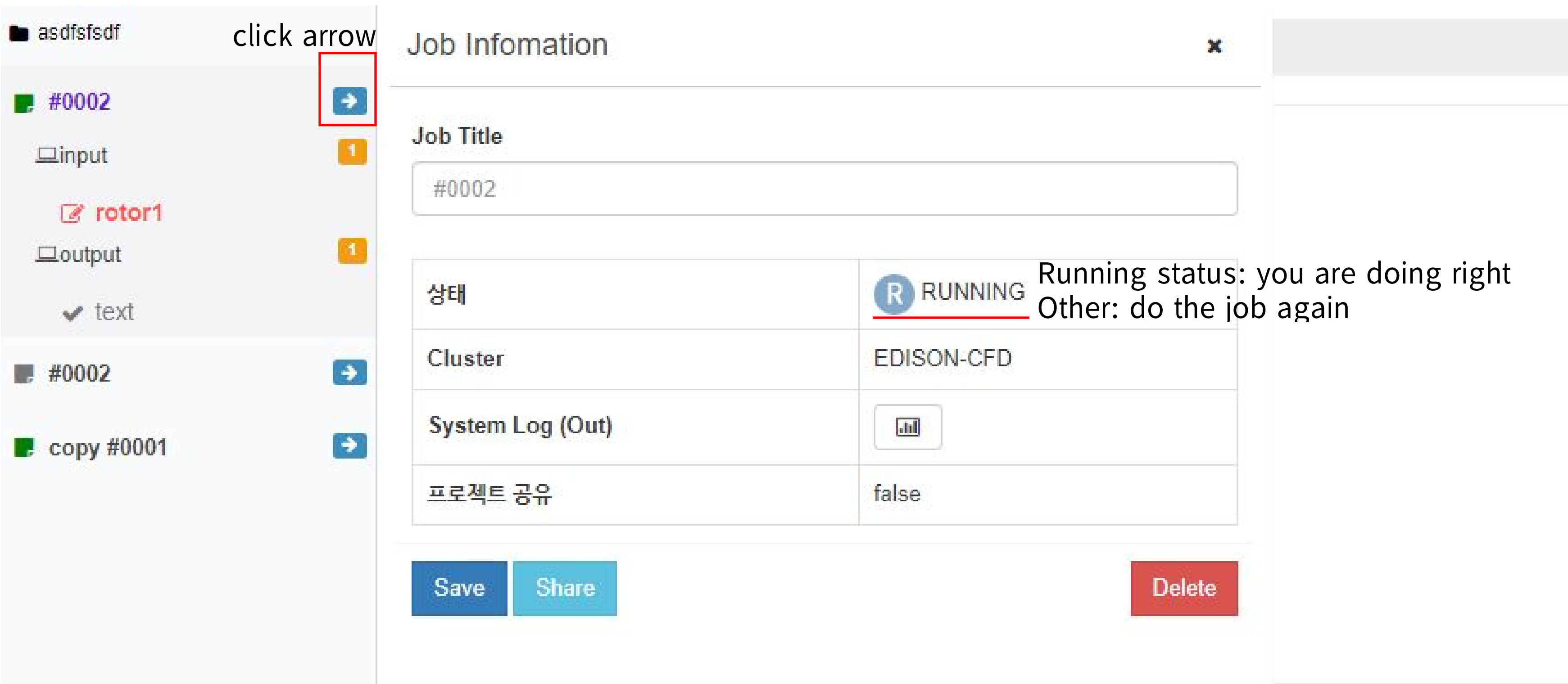

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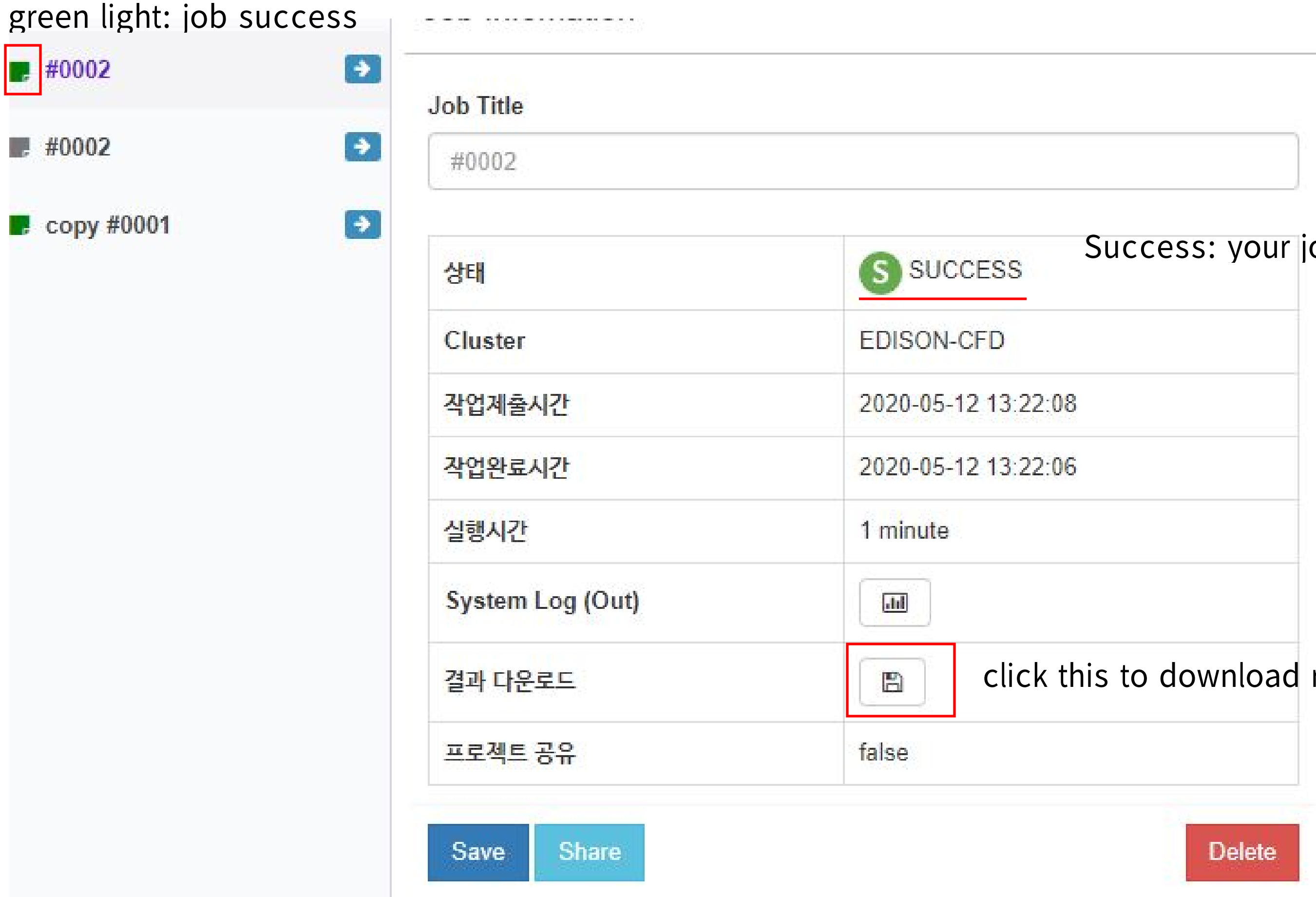
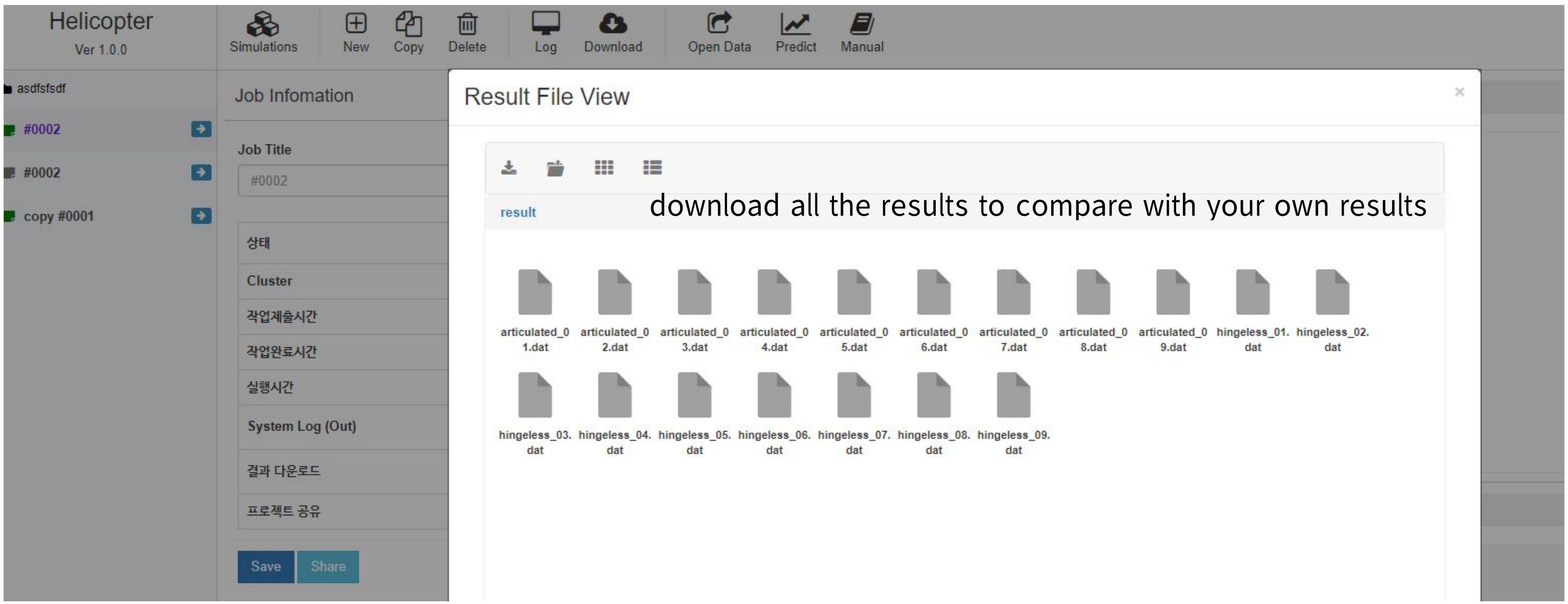
green light: job success



Success: your job is done

click this to download results

EDISON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Cluster
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작업완료시간
실행시간
System Log (Out)
결과 다운로드
프로젝트 공유
Save
Share
Result File View
result
download all the results to compare with your own results
articulated_01.dat
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articulated_03.dat
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hingeless_05.dat
hingeless_06.dat
hingeless_07.dat
hingeless_08.dat
hingeless_09.dat

❖ Open 'articulated_01~09.dat'

articulated_01.dat | articulated_05.dat | articulated_06.dat | articulated_07.dat | articulated_08.dat | articulated_09.dat | articulated_02.dat | articulated_03.dat | articulated_04.dat

1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
0.000	1.518	4.921	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

First mode **Second mode** **Third mode** **1/rev** **9/rev**

❖ Open 'hingeless_01~09.dat'

articulated_01.dat | articulated_05.dat | articulated_06.dat | articulated_07.dat | articulated_08.dat | articulated_09.dat | articulated_02.dat | articulated_03.dat | articulated_04.dat | hingeless_01.dat

1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
0.346	2.170	6.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

First mode **Second mode** **Third mode** **1/rev** **9/rev**

EXAMPLE) hingeless09.dat: results at 1/rev nominal frequency

1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1.710	4.588	9.019	1.600	3.200	4.800	6.400	8.000	9.600	11.200	12.800	14.400

these numbers can be used to plot the rotating frequency grids

EDISON

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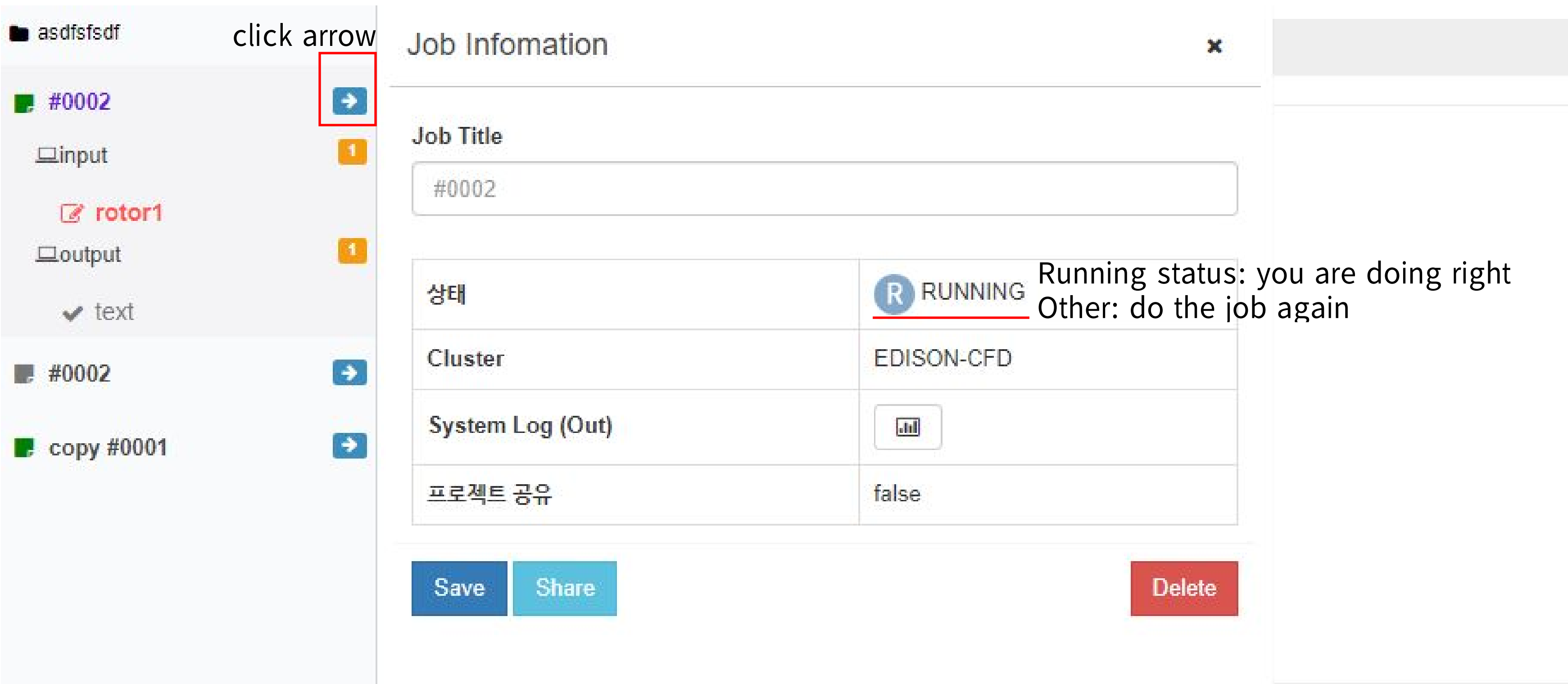

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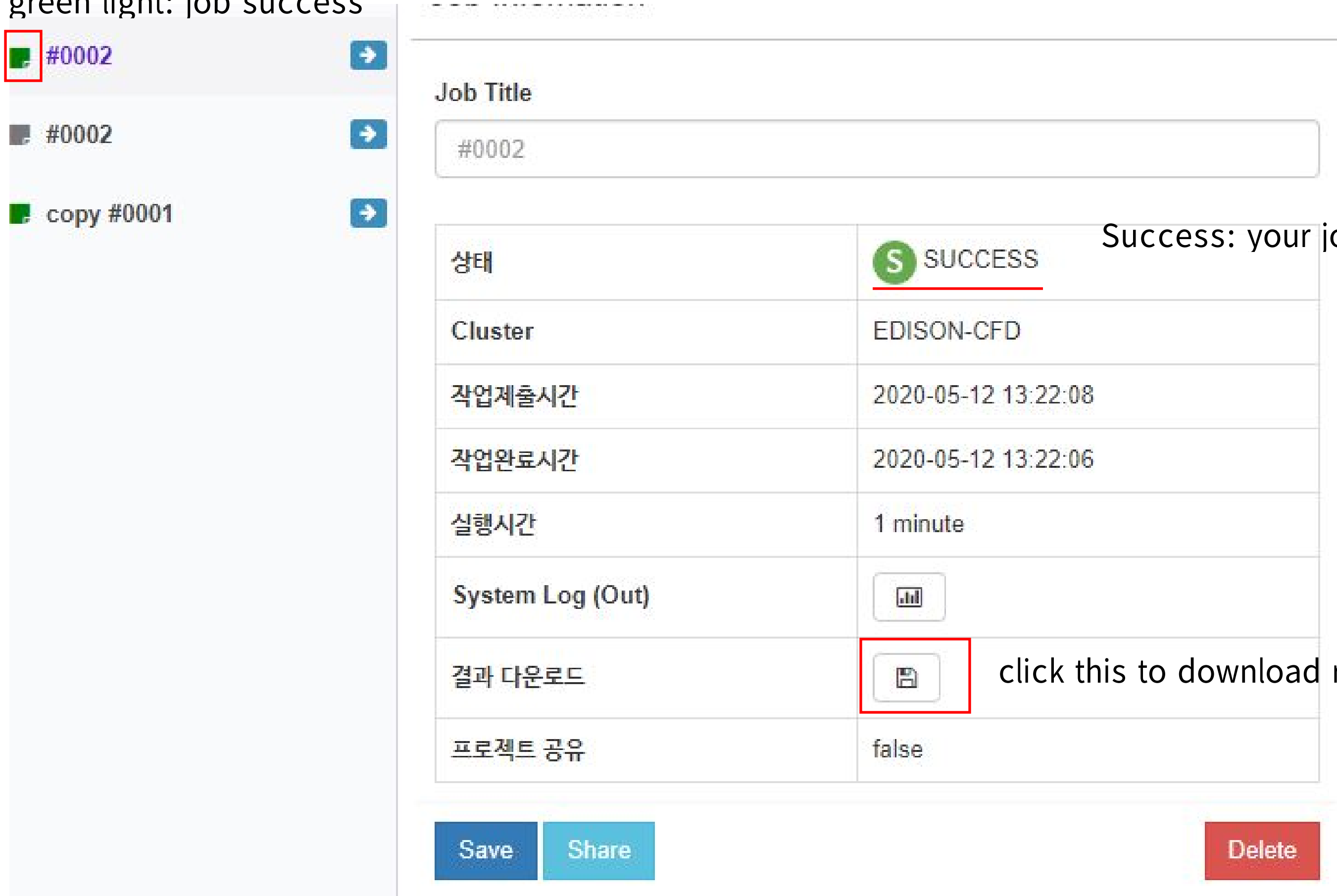

Success: your job is done

click this to download results

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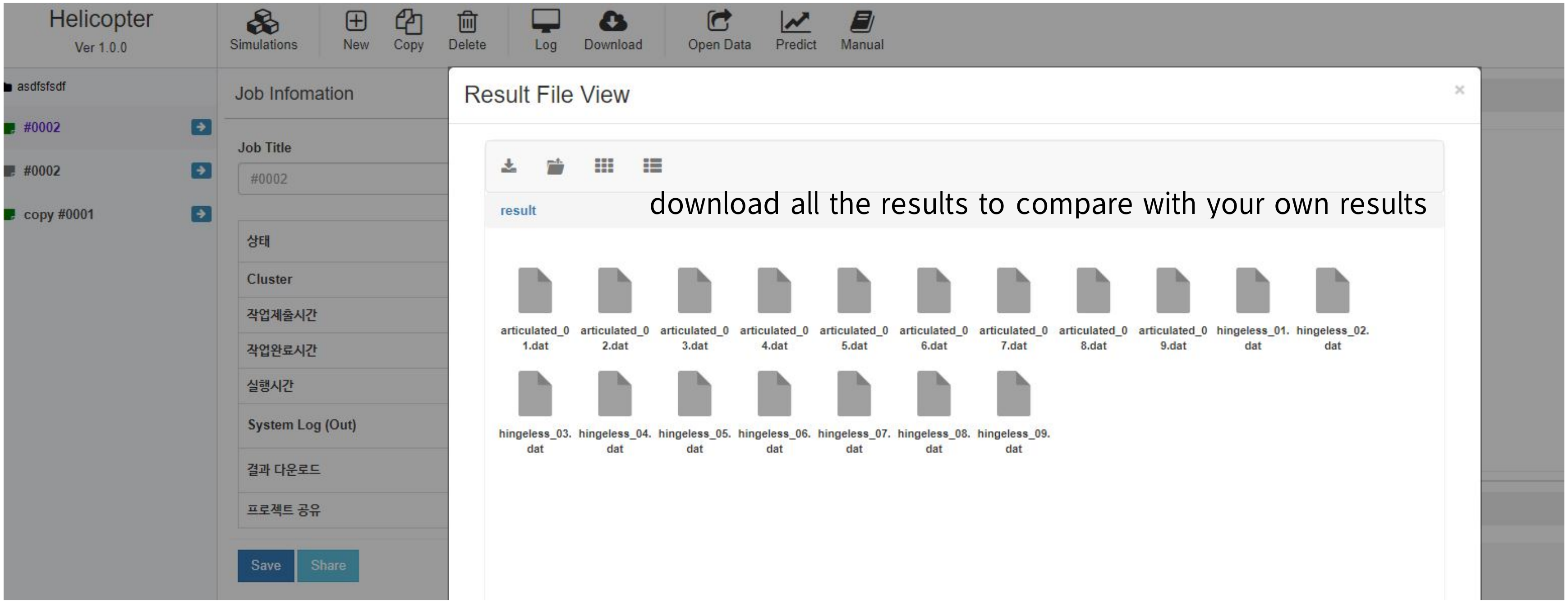

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Ver 1.0.0
Simulations
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Job Infomation
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실행시간
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EDISON

❖ Open ‘articulated_01~09.dat’

articulated_01.dat | articulated_05.dat | articulated_06.dat | articulated_07.dat | articulated_08.dat | articulated_09.dat | articulated_02.dat | articulated_03.dat | articulated_04.dat

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0.000	1.518	4.921	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

First mode **Second mode** **Third mode** **1/rev** **9/rev**

❖ Open ‘hingeless_01~09.dat’

articulated_01.dat | articulated_05.dat | articulated_06.dat | articulated_07.dat | articulated_08.dat | articulated_09.dat | articulated_02.dat | articulated_03.dat | articulated_04.dat | hingeless_01.dat

1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
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these numbers can be used to plot the rotating frequency grids