



**Seoul National University**  
**Active Aeroelasticity and Rotorcraft Lab.**

# Advanced Theory of Helicopter

## Term project #1

Finite Element Method (FEM) for structural analysis  
Prof. SangJoon Shin



**Term Project**

2020. 5. 12 / 15:30~16:00

Due date: 2020. 06. 19. 18:00

- 
- **Term Project**
  - **Theory**
  - **EDISON proejct**
  - **Conclusion**

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- **Term Project**

- Theory

- EDISON proejct

# Term project

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## ❖ **Make a complete code to calculate modes for rotating blade**

### ➤ **Use any computer languages**

- Your own codes must be included in your term project
- If any question or hints, e-mail to [goody147@snu.ac.kr](mailto:goody147@snu.ac.kr)

### ➤ **Use “EDISON project” to verify your results**

- Download “input.dat”
- If you change the input values, give you additional points.
- CPU run time is also important (+)
- If you find any bug, e-mail to [goody147@snu.ac.kr](mailto:goody147@snu.ac.kr) for debugging.

# Term project

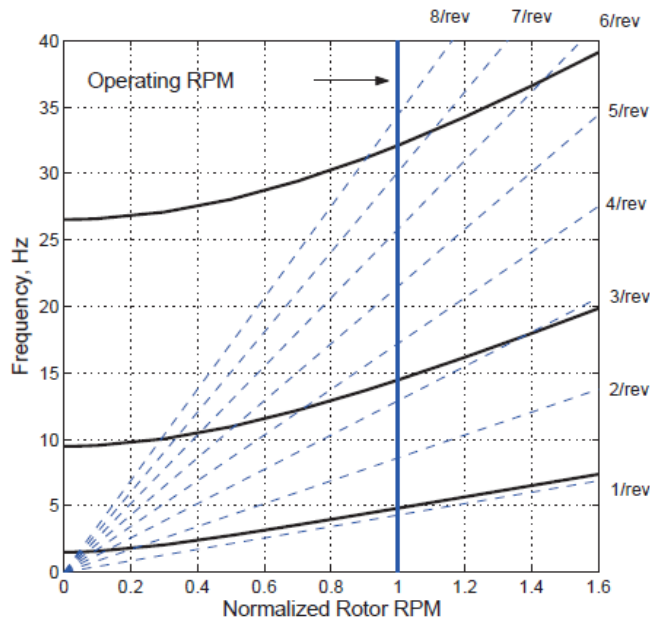
## ❖ Fan plot for rotating beams

### ➤ What is Fan plot for rotating beams?

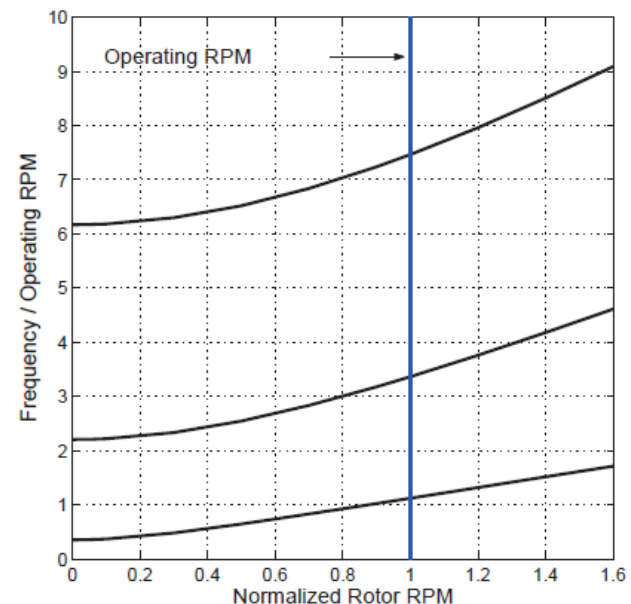
- The natural modes of a structure represent the unique ways it can vibrate in vacuum and without damping

### ➤ The natural frequencies

- Depends on mass(M) and stiffness(K), boundary conditions(B.C), and rotational speed( $\Omega$ )



(a) Fan plot in Hz



(b) Frequency plot non-dimensionalized with respect to operating RPM

# Term Project

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## ❖ Calculate the non-dimensional Fan-plot

### ➤ Approach

- Utilize the Finite Element Method(FEM) for structural analysis
- Natural frequencies( $f$ )  $\propto K/M$
- Two-dimensional beam

### ➤ Steps

- Discretize the rotating beam into a number of finite elements
  1. Develop the elemental properties
  2. Assemble the elemental properties
  3. Apply the constraints(B.C)

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- Term Project

- Theory

- EDISON proejct

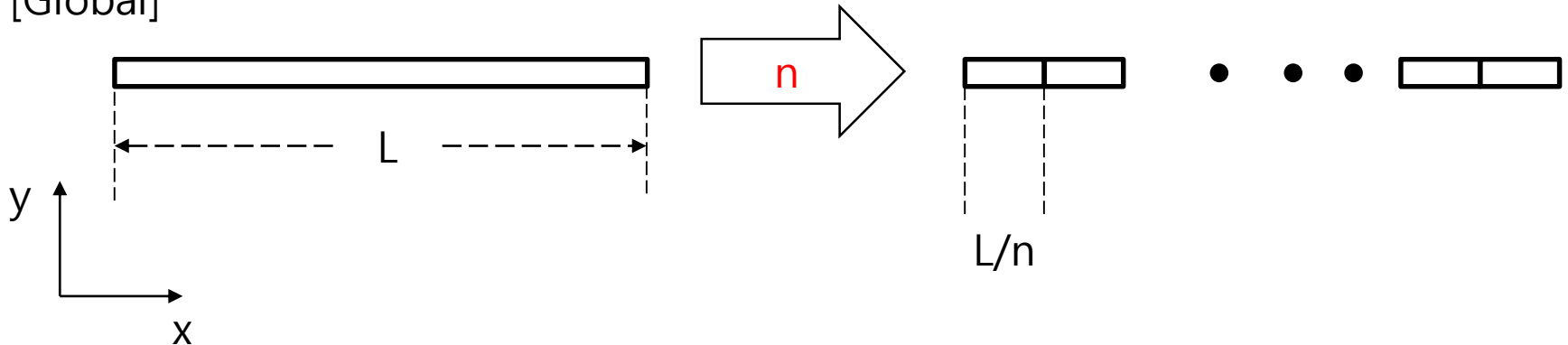
# Theory

## ❖ Concepts

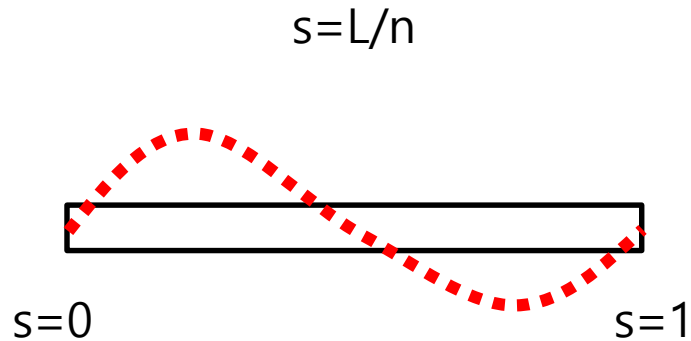
### ➤ Original geometry

- Discretize the rotating beam into a number of finite elements

[Global]



[Local]



$$w(s) = \sum_{i=1}^n H_i(s) q_i(t)$$

$H_i(s)$ : interpolation function  
 $q_i(s)$ : displacements

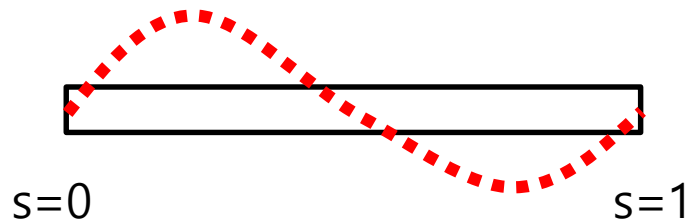


# Theory

## ❖ Concepts

- Assume interpolation function as a polynomial distribution w.r.t  $w$

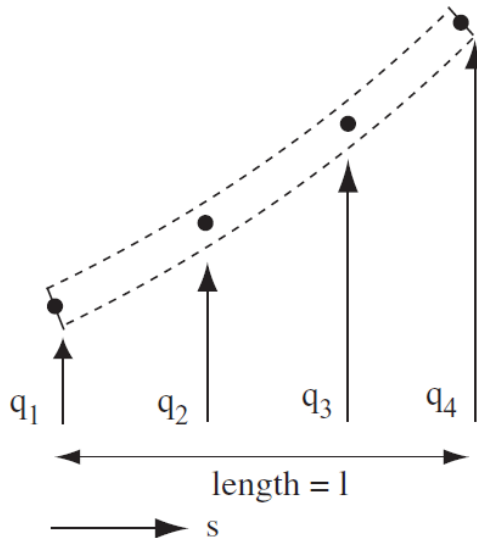
[Local]



$$w(s) = \sum_{i=1}^n H_i(s)q_i(t)$$

$H_i(s)$ : interpolation function

$q_i(s)$ : displacements



$$w(s, t) = \alpha_0 + s\alpha_1 + s^2\alpha_2 + s^3\alpha_3$$

$$w(0, t) = q_1 = \alpha_0$$

$$w(l/3, t) = q_2 = \alpha_0 + (l/3)\alpha_1$$

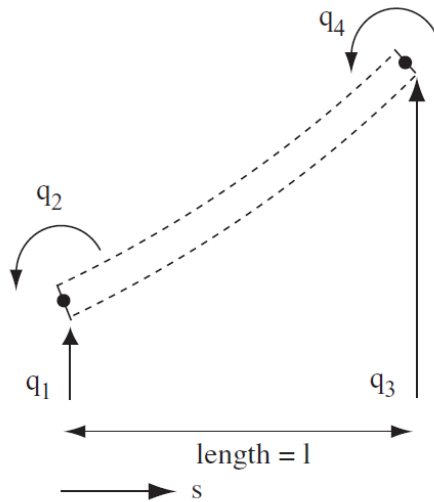
$$w(2l/3, t) = q_3 = \alpha_0 + (2l/3)\alpha_1 + (2l/3)^2\alpha_2 + (2l/3)^3\alpha_3$$

$$w(l, t) = q_4 = \alpha_0 + l\alpha_1 + l^2\alpha_2 + l^3\alpha_3$$

# Theory

## ❖ Concepts

- Solving for  $\alpha_0 \sim \alpha_3$  in terms of  $q_1 \sim q_4$



(d) Nodal displacements and angles of the finite element

$$w(s, t) = \alpha_1 + s\alpha_2 + s^2\alpha_3 + s^3\alpha_4$$

$$w(0, t) = q_1 = \alpha_1$$

$$\frac{dw}{ds}(0, t) = q_2 = \alpha_2$$

$$w(l, t) = q_3 = \alpha_1 + \alpha_2(l) + \alpha_3(l)^2 + \alpha_4(l)^3$$

$$\frac{dw}{ds}(l, t) = q_4 = \alpha_2 + 2\alpha_3(l) + 3\alpha_4(l)^2$$

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & (l)^2 & (l)^3 \\ 0 & 1 & 2l & 3(l)^2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix}$$

# Theory

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## ❖ Concepts

### ➤ General form of the interpolation functions

$$w(s) = \sum_{i=1}^n H_i(s)q_i(t)$$
$$\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{Bmatrix} = \begin{bmatrix} H_1 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & H_3 & 0 \\ 0 & 0 & 0 & H_4 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}$$

$$H_1 = 2\left(\frac{s}{l}\right)^3 - 3\left(\frac{s}{l}\right)^2 + 1$$

$$H_2 = \left(\frac{s}{l}\right)^2 - 2\left(\frac{s}{l}\right) + s$$

$$H_3 = -2\left(\frac{s}{l}\right)^3 + 3\left(\frac{s}{l}\right)^2$$

$$H_4 = \left(\frac{s}{l}\right)^2 - \left(\frac{s}{l}\right)$$

# Theory

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## ❖ Concepts

### ➤ The elemental energies using the Rayleigh-Ritz method

- Total potential energy  
= the kinetic energy ( $U_e$ ) + the strain energy ( $V_e$ ) + ~~virtual work ( $Q_e$ )~~
- The kinetic energy ( $U_e$ )

$$U_e = \frac{1}{2} \int_0^l m \dot{w}^2 ds = \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 m_{ij} \dot{q}_i \dot{q}_j$$

$$\text{where, } m_{ij} = \rho \int_0^l H_i H_j ds$$

- The strain energy ( $V_e$ )

$$V_e = \frac{1}{2} \int_0^l EI (\ddot{w})^2 ds + \frac{1}{2} \int_0^l T(s) (\dot{w})^2 ds = \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 k_{ij} q_i q_j$$

$$\text{where, } k_{ij} = EI \int_0^l \ddot{H}_i \ddot{H}_j ds + \int_0^l T(s) \dot{H}_i \dot{H}_j ds$$

# Theory

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## ❖ Concepts

### ➤ The elemental energies using the Rayleigh-Ritz method

- Total potential energy  
= the kinetic energy ( $U_e$ ) + the strain energy ( $V_e$ ) + ~~virtual work ( $Q_e$ )~~
- The kinetic energy ( $U_e$ )

$$U_e = \frac{1}{2} \int_0^l m \dot{w}^2 ds = \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 m_{ij} \dot{q}_i \dot{q}_j$$

$$\text{where, } m_{ij} = \rho \int_0^l H_i H_j ds$$

- The strain energy ( $V_e$ )

$$V_e = \frac{1}{2} \int_0^l EI (\ddot{w})^2 ds + \frac{1}{2} \int_0^l T(s) (\dot{w})^2 ds = \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 k_{ij} q_i q_j$$

$$\text{where, } k_{ij} = EI \int_0^l \ddot{H}_i \ddot{H}_j ds + \int_0^l T(s) \dot{H}_i \dot{H}_j ds$$

# Term project

## ❖ Rotating frequencies vs rotational speed (p.119)

### ➤ Use following values

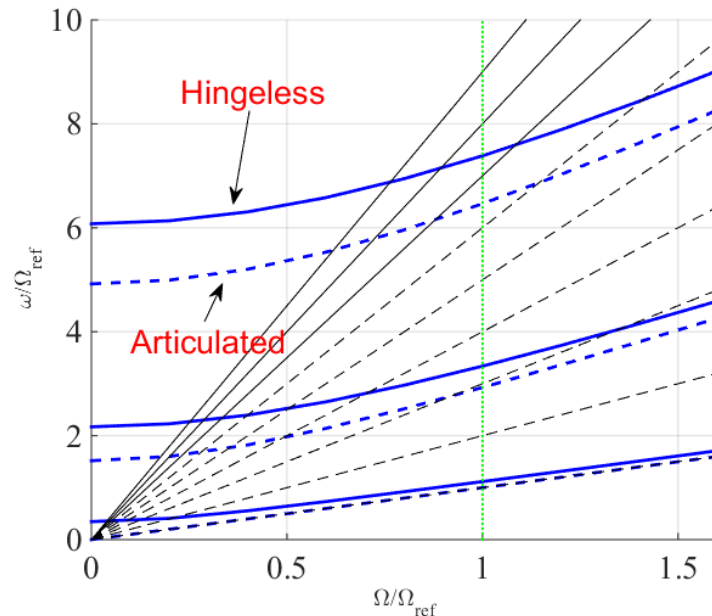
- $R = 8.2 \text{ m}$
- $EI = 4.225 \times 10^5$
- $M = 13 \text{ kg/m}$

### ➤ Fan-plot including both articulated and hingeless blade

- Plot first three modes w. r. t. non-dimensional rotating speed

### ➤ Any computer language are allowed

- MATLAB, FORTRAN90, C ...



# Information

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## ❖ Variables

### ➤ Geometrical variable

- Length (L) = 8.2 m
- Width (wid) = 0.4 m
- Height (h) = 0.04 m
- Area moment of inertia  $I_{zz} = 1$

### ➤ Material variable

- Young's modulus ( $E$ ) =  $4.225 \times 10^5$
- Shear modulus (G) =  $26.69 \times 10^9$
- Density ( $\rho$ ) = 13
- Nominal RPM (Non\_omega) = 260

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- Term Project

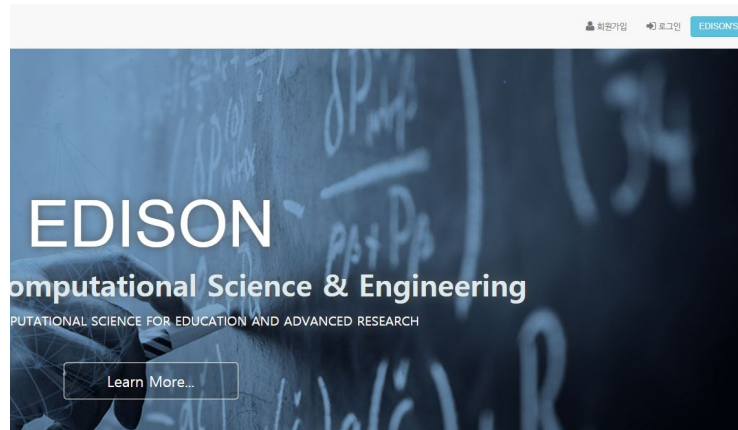
- Theory

- **EDISON** proejct



# EDISON

❖ [csd.edison.re.kr](http://csd.edison.re.kr)



Sign - up

Login

EDISON 통합검색 앱스토어 시뮬레이션 콘텐츠/데이터 교육 ABOUT My EDISON [Enter education](#)

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**항공역학**

한서대학교  
Prof. 박창환

**CHEM 분야**



**일반화학실험1 ,공학화학실험**

연세대학교  
Prof. 심은지

**CSD 분야**



**헬리콥터공학**

서울대학교  
Prof. 신상준

**CSD 분야**



**헬리콥터 고급 이론**

서울대학교  
Prof. 신상준


1 2 3

enter Advanced helicopter

# EDISON



❖ [csd.edison.re.kr](http://csd.edison.re.kr)



 강의 사이언스앱

**Helicopter**

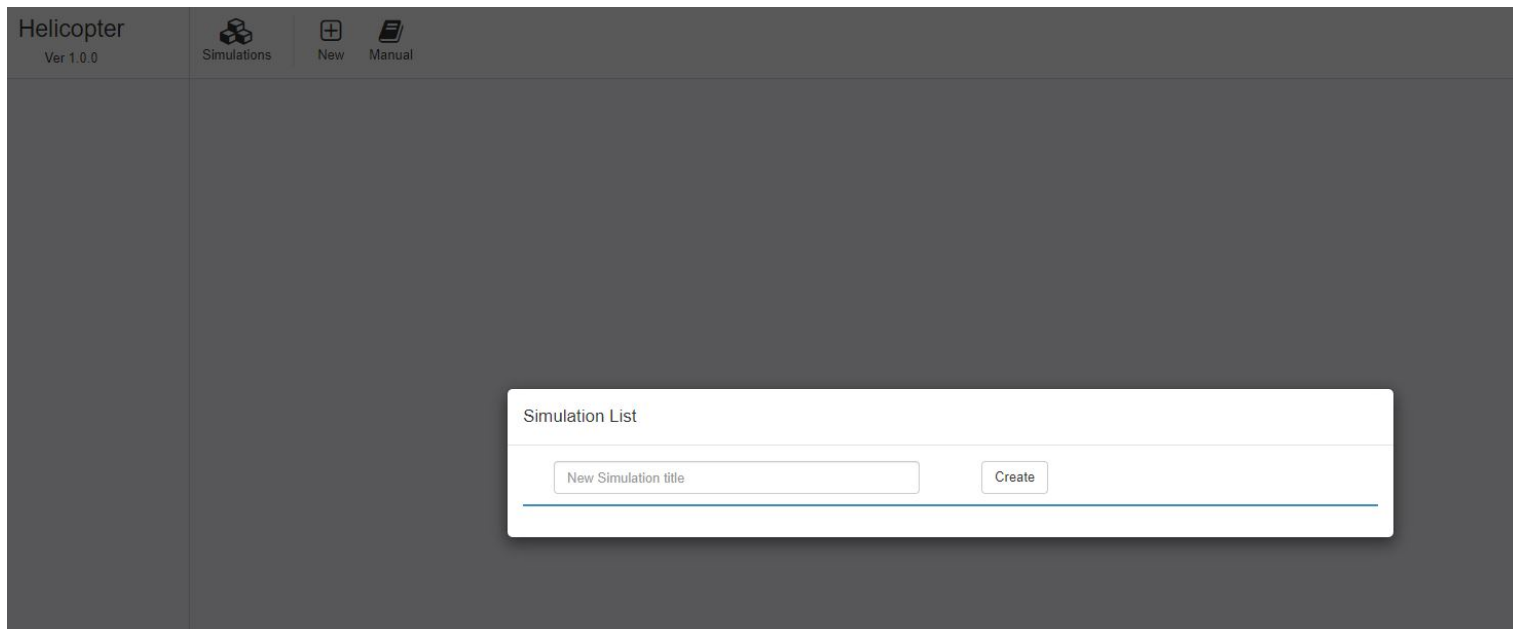
Ver 1.0.0  
서울대학교  
주현식

**RUN**  **MANUAL** 

Click run

# EDISON

❖ [csd.edison.re.kr](http://csd.edison.re.kr)



Enter simulation title

# EDISON

❖ [csd.edison.re.kr](http://csd.edison.re.kr)

click menu  
select Sample

Helicopter Ver 1.0.0

How to install — GetFEM++  
getfem.org/tutorial/install.html

Simulation

New Copy Delete Log Download Open Data Predict Manual

asdfsdf

copy #0001

input

rotor1

output

text

2018-05-17-05-19-29.711/input.dat

```
*****  
Graduate Courses Lectuer  
Advanced Theory of Helicopter  
Prof. Sang-Joon Shin(ssjoo@snu.ac.kr)  
*****  
Term Project #1  
2D Finite Element Method (FEM) for structural analysis  
Assistant: joohyunshig@snu.ac.kr  
*****  
*****  
W 0.4 % Width [m]  
H 0.04 % Height [m]  
A 1 % Second moment of the area  
L 8.2 % Length [m]  
Iz 1 % Integral Area moment of inertia  
E 4.225e5 % Young's modulus  
G 26.69e9 % Shear modulus  
Rho 13 % Mass Density  
nu 0.26 % Non omega  
*****
```

check variables (you may not change values)

# EDISON

❖ csd.edison.re.kr

click arrow

Job Information

Job Title

#0002

상태	<u>R</u> RUNNING
Cluster	EDISON-CFD
System Log (Out)	
프로젝트 공유	false



Running status: you are doing right  
Other: do the job again

Save Share Delete

# EDISON



## ❖ csd.edison.re.kr

green light: job success

-  #0002 
-  #0002 
-  copy #0001 

Job Title

#0002

상태	 <u>SUCCESS</u>	Success: your job is done
Cluster	EDISON-CFD	
작업제출시간	2020-05-12 13:22:08	
작업완료시간	2020-05-12 13:22:06	
실행시간	1 minute	
System Log (Out)		
결과 다운로드		click this to download results
프로젝트 공유	false	

Save

Share

Delete

# EDISON

The screenshot displays the Helicopter software interface. The top toolbar includes icons for Simulations, New, Copy, Delete, Log, Download, Open Data, Predict, and Manual. The left sidebar shows a project tree with folders like 'asdfsdf', '#0002', and 'copy #0001'. The main panel is divided into 'Job Information' and 'Result File View'. The 'Job Information' section includes fields for Job Title (#0002), State, Cluster, and various time-related metrics. The 'Result File View' window shows a list of files under the 'result' folder, with a prominent message: 'download all the results to compare with your own results'. The files are organized into two rows: the first row contains ten 'articulated\_0' files (1.dat to 9.dat) and two 'hingeless' files (hingeless\_01.dat, hingeless\_02.dat); the second row contains seven 'hingeless' files (hingeless\_03.dat to hingeless\_09.dat).

# EDISON

## ❖ Open 'articulated\_01~09.dat'

	1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1												
2	0.000	1.518	4.921	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3												

**First mode      Second mode      Third mode      1/rev      9/rev**

## ❖ Open 'hingeless\_01~09.dat'

	1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1												
2	0.346	2.170	6.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3												

**First mode      Second mode      Third mode      1/rev      9/rev**

EXAMPLE) hingeless09.dat: results at 1/rev nominal frequency

	1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1												
2	1.710	4.588	9.019	1.600	3.200	4.800	6.400	8.000	9.600	11.200	12.800	14.400

these numbers can be used to plot the rotating frequency grids









# EDISON

❖ csd.edison.re.kr

asdfsfsdf

click arrow

- #0002 
- input 
- rotor1 
- output 
- text

- #0002 
- copy #0001 

## Job Information

Job Title

#0002

상태	 RUNNING
Cluster	EDISON-CFD
System Log (Out)	
프로젝트 공유	false

Running status: you are doing right  
Other: do the job again

Save



Share

Delete

# EDISON




## ❖ csd.edison.re.kr

green light: job success

-  #0002 
-  #0002 
-  copy #0001 

Job Title

#0002

상태	 <u>SUCCESS</u>	Success: your job is done
Cluster	EDISON-CFD	
작업제출시간	2020-05-12 13:22:08	
작업완료시간	2020-05-12 13:22:06	
실행시간	1 minute	
System Log (Out)		
결과 다운로드		click this to download results
프로젝트 공유	false	

Save

Share

Delete

# EDISON

The screenshot displays the Helicopter software interface. The top toolbar includes icons for Simulations, New, Copy, Delete, Log, Download, Open Data, Predict, and Manual. The left sidebar shows a project tree with folders like 'asdfsdf', '#0002', and 'copy #0001'. The main panel is divided into 'Job Information' and 'Result File View'. The 'Job Information' section includes fields for Job Title (#0002), State, Cluster, and various time-related metrics. The 'Result File View' window shows a list of files under the 'result' folder, with a prominent message: 'download all the results to compare with your own results'. The files are organized into two rows: the first row contains ten 'articulated\_0' files (1.dat to 9.dat) and two 'hingeless' files (hingeless\_01.dat, hingeless\_02.dat); the second row contains seven 'hingeless' files (hingeless\_03.dat to hingeless\_09.dat).

Helicopter Ver 1.0.0

Simulations New Copy Delete Log Download Open Data Predict Manual

asdfsdf

#0002

#0002

copy #0001

Job Information

Job Title #0002

상태

Cluster

작업제출시간

작업완료시간

실행시간

System Log (Out)

결과 다운로드

프로젝트 공유

Save Share

Result File View

download all the results to compare with your own results

articulated\_0 1.dat articulated\_0 2.dat articulated\_0 3.dat articulated\_0 4.dat articulated\_0 5.dat articulated\_0 6.dat articulated\_0 7.dat articulated\_0 8.dat articulated\_0 9.dat hingeless\_01.dat hingeless\_02.dat

hingeless\_03.dat hingeless\_04.dat hingeless\_05.dat hingeless\_06.dat hingeless\_07.dat hingeless\_08.dat hingeless\_09.dat

# EDISON

## ❖ Open 'articulated\_01~09.dat'

	1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1												
2	0.000	1.518	4.921	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3												

**First mode      Second mode      Third mode      1/rev      9/rev**

## ❖ Open 'hingeless\_01~09.dat'

	1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1												
2	0.346	2.170	6.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3												

**First mode      Second mode      Third mode      1/rev      9/rev**

EXAMPLE) hingeless09.dat: results at 1/rev nominal frequency

	1ST	2ND	3RD	1/REV	2/REV	3/REV	4/REV	5/REV	6/REV	7/REV	8/REV	9/REV
1												
2	1.710	4.588	9.019	1.600	3.200	4.800	6.400	8.000	9.600	11.200	12.800	14.400

these numbers can be used to plot the rotating frequency grids