Mechanics of Materials and Lab.

Homework #6 (Shear in Bending & Mohr's Circle)

Due 7th June (Th) 10am

Problem # 1

5.10-1 through 5.10-6 A wide-flange beam (see figure) having the cross section described below is subjected to a shear force V. Using the dimensions of the cross section, calculate the moment of inertia and then determine the following quantities:

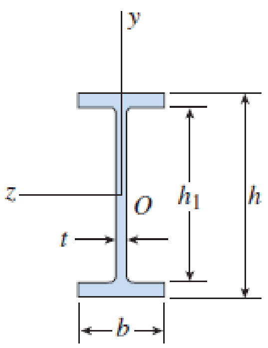
(a) The maximum shear stress τ_{max} in the web.

(b) The minimum shear stress τ_{min} in the web.

(c) The average shear stress τ_{aver} (obtained by dividing the shear force by the area of the web) and the ratio τ_{max}/τ_{aver}.

(d) The shear force V_{web} carried in the web and the ratio V_{web}/V.

(*Note:* Disregard the fillets at the junctions of the web and flanges and determine all quantities, including the moment of inertia, by considering the cross section to consist of three rectangles.)



PROBS. 5.10-1 through 5.10-6

5.10-1 Dimensions of cross section: b = 6 in., t = 0.5 in., h = 12 in., h_1 = 10.5 in., and V = 30 k.

5.10-2 Dimensions of cross section: b = 180 mm, t = 12 mm, h = 420 mm, h_1 = 380 mm, and V = 125 kN.

5.10-3 Wide-flange shape, W 8 × 28 (see Table E-1, Appendix E); V = 10 k.

5.10-4 Dimensions of cross section: b = 220 mm, t = 12 mm, h = 600 mm, h_1 = 570 mm, and V = 200 kN.

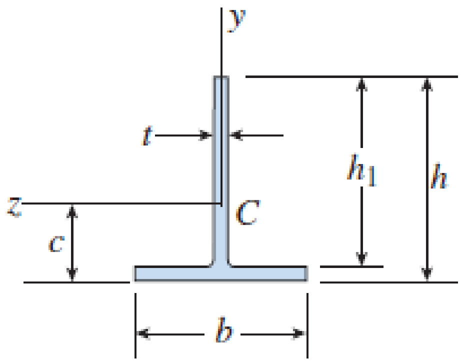
5.10-5 Wide-flange shape, W 18 × 71 (see Table E-1, Appendix E); V = 21 k.

5.10-6 Dimensions of cross section: b = 120 mm, t = 7 mm, h = 350 mm, h_1 = 330 mm, and V = 60 kN.

Problem #2

5.10-12 The T-beam shown in the figure has cross-sectional dimensions as follows: $b = 210$ mm, $t = 16$ mm, $h = 300$ mm, and $h_1 = 280$ mm. The beam is subjected to a shear force $V = 68$ kN.

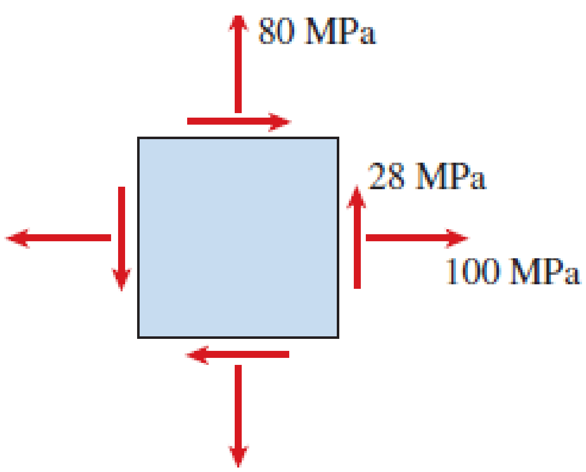
Determine the maximum shear stress τ_{max} in the web of the beam.



Problem #3

7.2-2 Solve the preceding problem for an element in *plane stress* subjected to stresses $\sigma_x = 100$ MPa, $\sigma_y = 80$ MPa, and $\tau_{xy} = 28$ MPa, as shown in the figure.

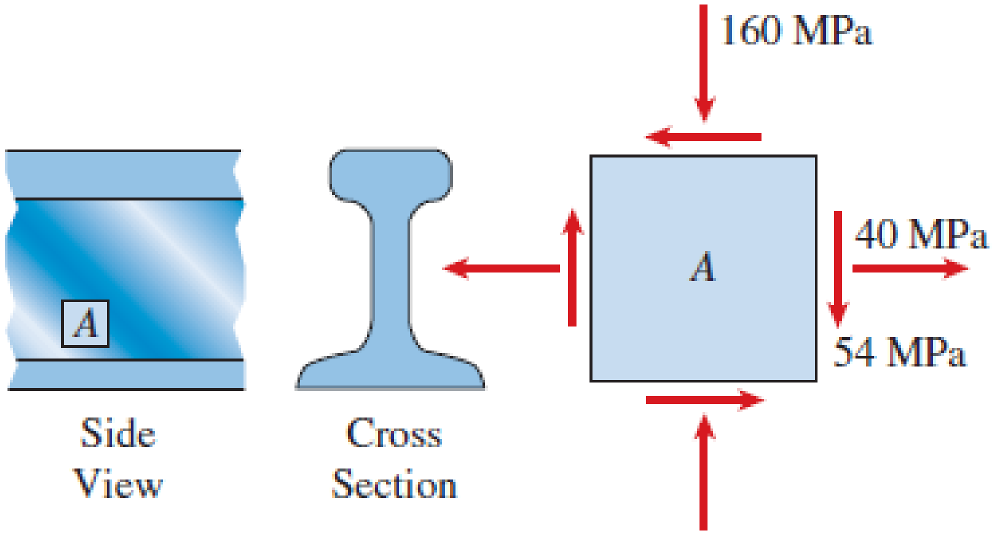
Determine the stresses acting on an element oriented at an angle $\theta = 30°$ from the x axis, where the angle θ is positive when counterclockwise. Show these stresses on a sketch of an element oriented at the angle θ.



Problem #4

7.2-4 The stresses acting on element *A* in the web of a train rail are found to be 40 MPa tension in the horizontal direction and 160 MPa compression in the vertical direction (see figure). Also, shear stresses of magnitude 54 MPa act in the directions shown.

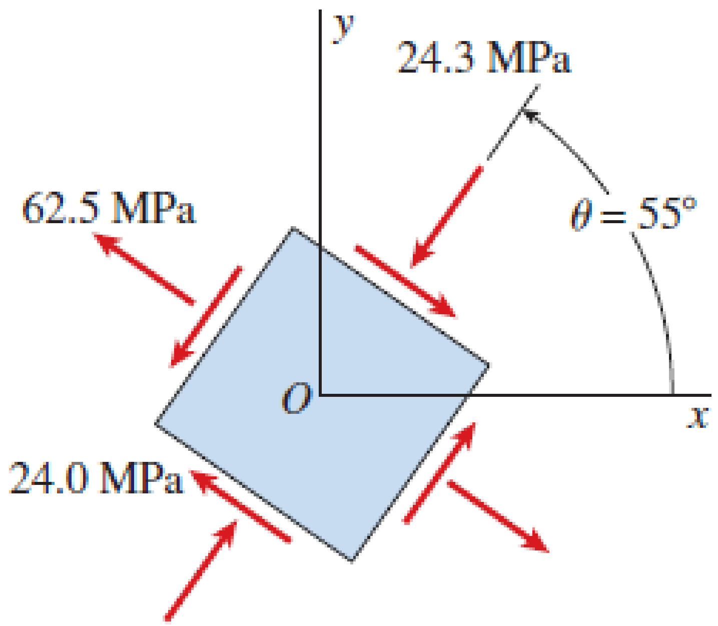
Determine the stresses acting on an element oriented at a counterclockwise angle of 52° from the horizontal. Show these stresses on a sketch of an element oriented at this angle.



Problem #5

7.2-15 An element in *plane stress* from the frame of a racing car is oriented at a known angle θ (see figure). On this inclined element, the normal and shear stresses have the magnitudes and directions shown in the figure.

Determine the normal and shear stresses acting on an element whose sides are parallel to the xy axes, that is, determine σ_x, σ_y, and τ_{xy}. Show the results on a sketch of an element oriented at $\theta = 0°$.



Problem #6

Derive below equation (Eq. 1) using plane stress conditions (Eq. 2 and 3)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(Eq. 1)

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

(Eq. 2)

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \cos 2\theta_{p_1} = \frac{\sigma_x - \sigma_y}{2R} \qquad \sin 2\theta_{p_1} = \frac{\tau_{xy}}{R}$$

(Eq. 3)