Viscous Flows
2019 Fall
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HW1 (due 10/1)

Solve the followings in Currie (main textbook).

1.1 Derive the continuity equation from first principles using an *infinitesimal control volume* of rectangular shape and having dimensions $(\delta x, \delta y, \delta z)$. Identify the net mass flow rate through each surface of this element as well as the rate at which the mass of the element is increasing. The resulting equation should be expressed in terms of the cartesian coordinates (x, y, z, t), the cartesian velocity components (u, v, w), and the fluid density ρ.

1.2 Derive the continuity equation from first principles using an *infinitesimal control volume* of cylindrical shape and having dimensions $(\delta R, R\delta\theta, \delta z)$. Identify the net mass flow rate through each surface of this element as well as the rate at which the mass of the element is increasing. The resulting equation should be expressed in terms of the cylindrical coordinates (R, θ, z, t), the cylindrical velocity components (u_R, u_θ, u_z), and the fluid density ρ.

1.3 Derive the continuity equation from first principles using an *infinitesimal control volume* of spherical shape and having dimensions $(\delta r, r\,\delta\theta, r\sin\theta\,\delta\omega)$. Identify the net mass flow rate through each surface of this element as well as the rate at which the mass of the element is increasing. The resulting equation should be expressed in terms of the cylindrical coordinates (r, θ, ω, t), the cylindrical velocity components $(u_r, u_\theta, u_\omega)$, and the fluid density ρ.

1.4 Obtain the continuity equation in cylindrical coordinates by expanding the vector form in cylindrical coordinates. To do this, make use of the following relationships connecting the coordinates and the velocity components in cartesian and cylindrical coordinates:

$$x = R\cos\theta$$

$$y = R\sin\theta$$

$$z = z$$

$$u = u_R\cos\theta - u_\theta\sin\theta$$

$$v = u_R\sin\theta + u_\theta\cos\theta$$

$$w = u_z$$

1.5 Obtain the continuity equation in spherical coordinates by expanding the vector form in spherical coordinates. Make use of the vector relationships outlined in Appendix A and follow the procedures used in Prob. 1.4.

1.6 Evaluate the radial component of the inertia term $(\mathbf{u} \cdot \nabla)\mathbf{u}$ in cylindrical coordinates using the following identities:

$$x = R \cos \theta$$

$$y = R \sin \theta$$

$$u\mathbf{e}_x + v\mathbf{e}_y = u_R\mathbf{e}_R + u_\theta\mathbf{e}_\theta$$

and any other vector identities from Appendix A as required. Here R and θ are cylindrical coordinates, u_R and u_θ are the corresponding velocity components, and $\mathbf{e}_R, \mathbf{e}_\theta$ are the unit base vectors.

1.7 Evaluate the radial component of the inertia term $(\mathbf{u} \cdot \nabla)\mathbf{u}$ in spherical coordinates by use of the vector identities given in Appendix A.

1.8 Start with the shear stress tensor τ_{ij}. Write out the independent components of this tensor in cartesian coordinates (x, y, z) using the cartesian representation (u, v, w) for the velocity vector. Specialize these expressions for the case of a monatomic gas for which the Stokes relation applies.

1.9 Write out the expression for the dissipation function, Φ, for the same conditions and using the same notation as defined in Prob. 1.8.

3.3 In cylindrical coordinates, the velocity components for a uniform flow around a circular cylinder are

$$u_R = U\left(1 - \frac{a^2}{R^2}\right)\cos\theta$$

$$u_\theta = -U\left(1 + \frac{a^2}{R^2}\right)\sin\theta$$

Here U is the constant magnitude of the velocity approaching the cylinder and a is the radius of the cylinder. If compressible and viscous effects are negligible, determine the pressure $p(R, \theta)$ at any point in the fluid in the absence of any body forces. Take the pressure far from the cylinder to be constant and equal to p_0.
Specialize the result obtained above to obtain an expression for the pressure $p(a, \theta)$ on the surface of the cylinder.