Viscous Flows

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HW2 (due 10/31)

Solve the followings in Currie (Ch. 7) (main textbook).

[1]

To explain the manner in which Couette flow is established, find the velocity distribution in a fluid that is bounded by two horizontal parallel surfaces in which everything is quiescent for $t < 0$ and for which the upper surface is impulsively set into horizontal motion with constant velocity U at time $t = 0$. (This can be done by obtaining the solution in its asymptotic form, corresponding to $t \to \infty$, then adding a separation of variables solution.)

[2]

A viscous fluid occupies the space between two stationary parallel horizontal surfaces defined by the lines $y = -1$ and $y = +1$. Initially, there is no motion in the fluid, but at time $t = 0$, a valve is opened and the fluid begins to move in the x direction under the pressure differential that exists between the ends of the conduit. The problem to be solved for the subsequent fluid velocity $u(y,t)$, subject to the appropriate boundary conditions, is the following:

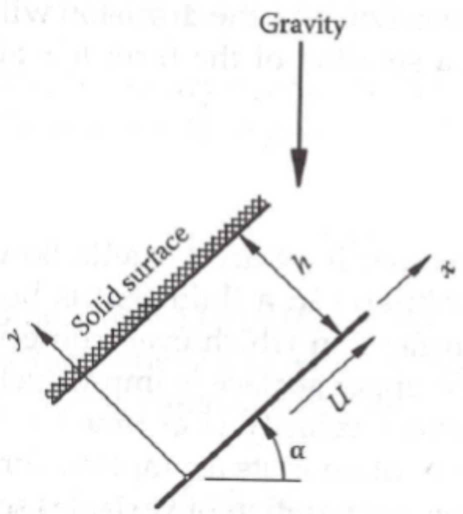
$$\frac{\partial u}{\partial t} = 2PH(t) + \frac{\partial^2 u}{\partial y^2} \quad \text{for } -1 \le y \le +1.$$

The velocity and the two independent variables have been made dimensionless in this equation. The constant P is the dimensionless value of the pressure gradient in the flow direction for $t > 0$ (i.e., in the x direction), and $H(t)$ is the Heaviside function (unit step function). Obtain the solution to the foregoing problem that is valid for all times $t > 0$.

[3]

A moving belt is inclined at an angle α to the horizontal. The lower end of this belt is immersed in a pool of liquid, and the belt drags some of the liquid with it as it moves upward and out of the liquid. The liquid may be assumed to be viscous but incompressible.

(a) Using the configuration shown in the following figure, solve the Navier–Stokes equations for the following quantities:



(i) The velocity distribution in the liquid layer

(ii) The volumetric flow rate of liquid in the x direction per unit width

(iii) The angle α for which the volumetric flow rate is zero

[4]

For Poiseuille flow through an elliptic pipe of semiaxes a and b, find the ratio b/a that gives the maximum flow rate for a given flow area and a given pressure gradient.

For a given pressure gradient, find the ratio of the discharge from an elliptic pipe to that from a circular pipe that has the same flow area. Evaluate this ratio for the specific value $b/a = 4/3$.

[5]

Two concentric circular cylinders enclose a viscous fluid. If the inner cylinder is at rest and the outer cylinder rotates at a constant angular velocity, calculate the torque required to rotate the outer cylinder and that required to hold the inner cylinder at rest.

[6]

Using the solution for flow between concentric rotating circular cylinders, deduce the velocity distribution created by a circular cylinder that is rotating in a fluid of infinite extent that is otherwise at rest. Compare this result with that for a line vortex of strength $\Gamma = 2\pi R_i^2 \omega_i$ in an inviscid fluid that is at rest at infinity.

[7]

For potential flow due to a line vortex, the vorticity is concentrated along the axis of the vortex. Thus, the problem to be solved for the decay of a line vortex with time due to the viscosity of the fluid is as follows:

$$\frac{\partial \omega}{\partial t} = \upsilon \nabla^2 \omega$$

$$\omega(R,0) = 0 \quad \text{for} \quad R > 0$$

$$\int_0^\infty \omega(R,t) 2\pi R \, \mathrm{d}R = \Gamma \quad \text{for} \quad t \geq 0.$$

Here, $\omega(R, t)$ is the vorticity, and the total circulation associated with the vortex for any time $t \geq 0$ is Γ. Look for a similarity solution to this problem of the following form:

$$\omega(R,t) = \frac{\Gamma}{2\pi \upsilon t} f\left(\frac{R}{\sqrt{\upsilon t}}\right).$$

Thus, obtain expressions for the velocity $u_\theta(R, t)$ and the pressure $p(R, t)$ in the fluid.

[8]

The following flow field satisfies the continuity equation everywhere except at $R = 0$, where a singularity exists:

$$u_R = -aR$$

$$u_\theta = \frac{K}{R}$$

$$u_z = 2az.$$

Show that this flow field also satisfies the Navier–Stokes equations everywhere except at $R = 0$, and find the pressure distribution in the flow field.

Modify the foregoing expressions to the following:

$$u_R = -aR$$

$$u_\theta = \frac{K}{R} f(R)$$

$$u_z = 2az.$$

Determine the function $f(R)$ such that the modified expression satisfies the governing equations for a viscous, incompressible fluid and such that the original flow field is recovered for $R \to \infty$.