Homework #2

(Problems in Currie Ch. 2 and 3)

2.5 Consider the two-dimensional velocity distribution defined as follows:

$$u = -\frac{x}{x^2 + y^2} \qquad v = \frac{y}{x^2 + y^2}$$

Determine the circulation for this flow field around the following contour by integrating around it counterclockwise:

$$\begin{aligned} -1 \le x \le +1 \qquad & y = -1 \\ -1 \le y \le +1 \qquad & x = +1 \\ -1 \le x \le +1 \qquad & y = +1 \\ -1 \le y \le +1 \qquad & x = -1 \end{aligned}$$

2.6 A particular three-dimensional flow field has the following velocity components:

$$u = 9x^2 + 2y \qquad v = 10x \qquad w = -2yz^2$$

(a) Using the same contour as defined in Prob. 2.5 on the plane $z = 5$, determine the circulation for the given flow field.

(b) Calculate the vorticity vector for the given flow field at any point (x, y) on the plane $z = 5$.

(c) Using the value obtained in (b) for the velocity vector $\boldsymbol{\omega}$ on the plane $z = 5$, evaluate the following integral:

$$\int_A \boldsymbol{\omega} \cdot \mathbf{n} \, dA$$

where A is the area of the rectangle defined in (a) and $\mathbf{n}$ is the unit normal to that area in the positive z direction. Compare the result obtained in (c) with that obtained in (a).

2.8 Calculate the vorticity at any point (R, θ) for each of the following two-dimensional flow fields:

(a) $u_R = 0, u_\theta = \omega R$.

(b) $u_R = 0, u_\theta = \Gamma/2\pi R$.

In the above, R and θ are cylindrical coordinates while ω and Γ are constants.

3.2 Show that, for an incompressible fluid, the following identity holds between the velocity vector $\mathbf{u}$ and the vorticity vector $\boldsymbol{\omega}$:

$$\boldsymbol{\nabla} \cdot [(\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u}] = \frac{1}{2}\nabla^2(\mathbf{u} \cdot \mathbf{u}) - \mathbf{u} \cdot (\nabla^2 \mathbf{u}) - \boldsymbol{\omega} \cdot \boldsymbol{\omega}$$

3.3 In cylindrical coordinates, the velocity components for a uniform flow around a circular cylinder are

$$u_R = U\left(1 - \frac{a^2}{R^2}\right)\cos\theta$$

$$u_\theta = -U\left(1 + \frac{a^2}{R^2}\right)\sin\theta$$

Here U is the constant magnitude of the velocity approaching the cylinder and a is the radius of the cylinder. If compressible and viscous effects are negligible, determine the pressure $p(R, \theta)$ at any point in the fluid in the absence of any body forces. Take the pressure far from the cylinder to be constant and equal to p_0.
Specialize the result obtained above to obtain an expression for the pressure $p(a, \theta)$ on the surface of the cylinder.