Viscous Flows
2019 Fall
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# HW3 (due 11/21)

[1]

Using the Stokes solution for uniform flow over a sphere, integrate the pressure around the surface of the sphere to establish the pressure drag that acts on the sphere. Hence deduce what portion of the total Stokes drag is due to the pressure distribution and what portion is due to the viscous shear on the surface of the sphere.

[2]

A liquid drop whose viscosity is $\mu'$ moves slowly through another liquid of viscosity $\mu$ with velocity $U$. The shape of the drop may be taken to be spherical, and the motion to be sufficiently slow that inertia of the fluid may be neglected. The boundary conditions at the surface of the drop are that the velocity and the tangential stresses in the two fluids are the same.

Show that the solution to the problem above exists in which the pressure inside the drop is proportional to $x$ and that outside the drop is proportional to $x/r^3$. From the solution, calculate the drag of the drop and show that it is smaller than that for a rigid sphere of the same size, the drag ratio being:

$$\frac{1 + 2/3(\mu/\mu')}{1 + \mu/\mu'}$$

[3]

The solution to the boundary layer equations corresponding to flow in a convergent channel resulted in the following ordinary differential equation:

$$f''' + 1 - (f')^2 = 0$$

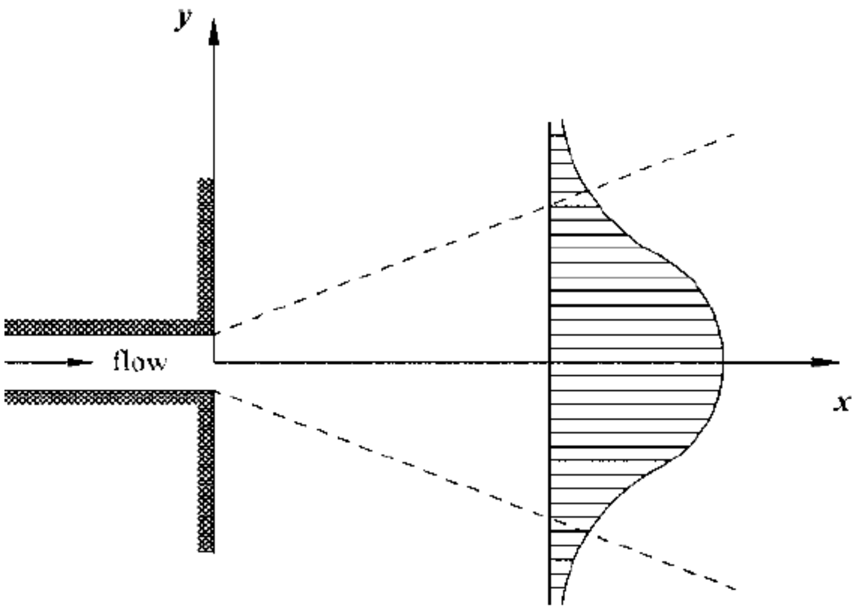
Show that this third-order, nonlinear, ordinary differential equation may be integrated to give

$$f'(\eta) = 3\ \tanh^2\left[\frac{\eta}{\sqrt{2}} + 1.146\right] - 2$$

[4]

Figure 9.10 illustrates a two-dimensional jet entering a reservoir that contains a stationary fluid. A solution is sought to the laminar boundary layer equations for this situation. Assuming that there is no pressure gradient along the jet, look for a similarity solution for the stream function of the following form:

$$\psi(x, y) = 6\alpha\nu x^{1/3} f(\eta)$$




[5]

Use the momentum integral and the velocity profile

$$\frac{u}{U} = a + b\frac{y}{\delta}$$

to evaluate the boundary-layer thicknesses $\delta$, $\delta^*$, and $\theta$ and the surface shear stress $\tau_0$ for flow over a flat surface.