**409.319A**

# *Aero Thermo Hydro Engineers Nexus Application*

Seoul National University
Fall 2018

***Problem Set 3***

**1. Fluid Kinematics Fundamentals** **10%**

Fluid kinematics depicts the motion of fluids without considering the forces and moments that cause the motion.

A. Introduce several kinematic concepts related to flowing fluids.
B. Discuss the material derivative and its role in transforming the conservation equations from the Lagrangian description of fluid flow following a fluid particle to the Eulerian description of fluid flow pertaining to a flow field.
C. Discuss various ways to visualize flow fields such as streamlines, streaklines, pathlines, timelines, and optical methods schlieren and shadowgraph.
D. Describe three ways to plot flow data such as profile plots, vector plots, and contour plots.
E. Explain the four fundamental kinematic properties of fluid motion and deformation—rate of translation, rate of rotation, linear strain rate, and shear strain rate.
F. Discuss the concepts of vorticity, rotationality, and irrotationality in fluid flows. Discuss the Reynolds transport theorem (RTT), emphasizing its role in transforming the equations of motion from those following a system to those pertaining to fluid flow into and out of a control volume.
G. Explain the analogy between material derivative for infinitesimal fluid elements and RTT for finite control volumes resorting to physical illustrations and mathematical expressions.

**2. Gas Balloon** **10%**

A gas-filled balloon before launching is taut, and has a volume of 10,000 $ft^3$ filled with helium gas (molecular weight 4, as compared with molecular weight 29 for air). The dead weight of the balloon is 150 lbm, the crew weight 300 lbm, and 150 lbm of ballast are carried in the form of sand bags.

A. What is the force which must be withstood by the anchor rope?
B. If the anchor rope is suddenly cut, what will be the initial upward acceleration of the balloon, neglecting the surrounding air motion?
C. Since the balloon material is thin and unable to withstand significant pressure differences, helium must be allowed to escape through a vent so that the internal pressure equals the external. Assuming sufficient heat conduction to make the helium temperature always equal the outside air temperature, to what altitude will the balloon rise if no ballast is thrown overboard?
D. What is the maximum altitude to which the balloon can rise?
E. How can the daring astronauts get down from this altitude?

**3. Steady Flow** **10%**

In many important applications the flow is steady and the control volume is fixed.

A. Express the mass conservation if the fluid enters normal to the inlet area $A_1$ and exits normal to the exit area $A_2$.
B. Assume $V_1$ is a parabolic profile in a pipe, so that $V_1 = V_{max}(1 - r^2/r_1^2)$ where $r_1$ is the radius of the pipe. Sketch the velocity profile, and determine $V_1$ if $V_{max} = 20$ fps.
C. An incompressible fluid exits through two areas, $A_2$ and $A_3$. Assuming uniform profiles, write the simplified continuity equation.
D. A control volume between the two pistons is divided into three parts: $V_1$, $V_2$, and $V_3$, with the areas separating $V_3$ from $V_1$ and $V_2$ located at fixed distances $l_1$ and $l_2$, respectively. Find the integral form of the continuity equation by applying Leibniz' rule to the time-rate-of change term after expressing the integral in appropriate form for the three volumes which make up the control volume. Assume all quantities to be functions of $x$ and $t$.

**4. Newton's Laws of Motion** **10%**

A fundamental requisite of any correct mathematical expression is that the quantities between which equality is stated to exist be dimensionally as well as numerically equivalent. That is, an equation must be dimensionally homogeneous if the physical condition that it expresses is to be generally true.

A. All mechanics phenomena represent an application of Newton's laws of motion describable in terms of force, mass, length and time. Find a dimensionless group pursuant to Newton's second law.
B. As a typical example of the homogeneity test for the dimensional correctness of an equation, express the velocity in terms of pressure drop and fluid density. Prove its homogeneity in dimensional terms.
C. List as many dimensions of quantities describing the boundary, flow, and fluid characteristics as you can in terms of geometric, kinematic and dynamic symbols and fundamental dimensions.
D. Delineate the three groups of those describing the boundary geometry, the kinematics, and the dynamics of the flow pattern.
E. Give a dimensionless pressure-velocity relationship, and discuss its engineering practicality.

**5. More Interesting Kinematics** **10%**

View https://www.youtube.com/watch?v=mdN8OOkx2ko for Eulerian and Lagrangian descriptions of fluid flow and write up a one full page essay on fluid kinematics.

**6. Tire with a Puncture** **10%**

If a car tire with a slow puncture takes 72 hours to decrease from its recommended pressure of 35 psi to 22 psi at which the low pressure light activates on the instrument panel, calculate the area $A$ of the hole given the mass flow rate from the tire as $0.66pA/(RT)^{1/2}$ where $R$ is the gas constant, $T$ is the absolute temperature and $p$ is the tire pressure. You may assume that the change in volume of the tire is negligible.

**7. Pitot Tube** **10%**

A pitot tube consists of two concentric cylinders, the inner one of which is open to oncoming air that stagnates in the cylinder. Thus, the stagnation pressure can be measured at the end of this inner cylinder. The outer cylinder is closed to oncoming air but has several holes in its surface that permit measurement of static pressure from the flow passing these holes. If temperature is measured with a thermocouple, the density can be calculated from an equation of state. It is desired to find the flow speed.

A. Sketch a pitot tube of your own design.

B. Write Bernoulli's equation between the locations 1 at the stagnation point and 2 at the port for static pressure measurement, and indicate the points in the figure.

C. Determine the incoming flow speed.

D. Notice that you have had to use two different streamlines to obtain this result, a fact that is often ignored for this problem. Are the results correct then? If so, what kind of flow condition did you have to assume?

E. While this may not necessarily be completely accurate, it does provide a reasonable approximation in the present case. Consider this in detail for both the inviscid approximation and the actual, physical viscous cases. Note that it is also common practice to make automatic corrections to pitot tube readouts of speed to at least partially account for the approximate nature of the analysis.

**8. Air Hockey Table** **10%**

You are designing an air hockey table. The table is 3.0 6.0 ft in area, with 1/16-in-diameter holes spaced every inch in a rectangular grid pattern (2592 holes total). The required jet speed from each hole is estimated to be 50 ft/s. Your job is to select an appropriate blower which will meet the requirements. Estimate the volumetric flow rate in $\mathrm{ft}^3/\mathrm{min}$ and pressure rise in $\mathrm{lb_f/in^2}$ required of the blower. You may assume that the air is stagnant in the large volume of the manifold under the table surface, and neglect any frictional losses.

**9. Bernoulli's Equation Applications** **10%**

A. A large pressurized tank filled with air discharges into the atmosphere. The flow path is a short and frictionless smooth pipe connected to a discharge nozzle. Find (a) the flow rate of air and (b) pressure in the pipe for the given data. Ignore all frictional losses, including head losses at the entrance to the pipe, at the bend, and at the nozzle. $P_{\mathrm{Tank}}$ = 110 kPa, $D_{\mathrm{Pipe}}$ = 0.040 m, $d_{\mathrm{Nozzle}}$ = 0.015 m; $P_{\mathrm{atm}}$ = 101 kPa, $T_{\mathrm{atm}}$ = 293 K.

B. A siphon (an inverted U-tube) is used to steadily withdraw water from a large reservoir. The top of the siphon is 1.5 m higher than the surface of the water in the reservoir and the discharge side of the siphon is 8 m below the water surface. Ignore all frictional losses in the siphon. Find (a) the mass flow rate of water discharged to the atmosphere and (b) the pressure at the top of the siphon. Water is at $T$ = 300 K and 1 atm. The diameter of the siphon tube is 5 cm.

C. A pipe is connected vertically to the discharge side of a pump. The top of the pipe is a short horizontal segment connected to a nozzle. The vertical length of the pipe (i.e., from the pump discharge to the horizontal segment) is 12 ft long and the pipe diameter is 4 in. The nozzle discharges water to the atmosphere at a velocity of 65 ft/s. Ignore all frictional losses and find the required pressure at the pump discharge.

D. A turbine is operating at 150 ft below the surface of a lake. Flow rate of water through the turbine is 100 $\mathrm{ft}^3$/s. The discharge pipe is located 10 ft above the turbine. In the discharge pipe, where velocity is 25 ft/s, pressure is measured as 12 psig. Find the maximum power developed by this turbine.

E. An open circuit wind tunnel draws in sea level standard air and accelerates it through a contraction into a 1 m by 1 m test section. A differential transducer mounted in the test section wall measures a pressure difference of 45 mm of water between the inside and outside. Estimate (a) the test section velocity and (b) the absolute pressure on the front nose of a small model mounted in the test section.

**10. Hydrodynamica** **10%**

*Hydrodynamica* is a book published by Daniel Bernoulli in 1738. The title of this book eventually christened the field of fluid mechanics as hydrodynamics. The book deals with fluid mechanics and is organized around the idea of conservation of energy, as received from Christiaan Huygens' formulation of this principle. The book describes the theory of water flowing through a tube and of water flowing from a hole in a container. In doing so, Bernoulli explained the nature of hydrodynamic pressure and discovered the role of loss of *vis viva* in fluid flow, which would later be known as the Bernoulli principle. The book also discusses hydraulic machines and introduces the notion of work and efficiency of a machine. In the tenth chapter, Bernoulli discussed the first model of the kinetic theory of gases. Assuming that heat increases the velocity of the gas particles, he demonstrated that the pressure of air is proportional to kinetic energy of gas particles, thus making the temperature of gas proportional to this kinetic energy as well. A PDF file is being uploaded for your eyes only.

A. The last 12 pages of this monumental treatise contain numerous excellent sketches of flow patterns related to his frictionless relation. Redraw some of those and explain what he might have attempted to deliberate.

B. One, however, seems physically misleading. Can you tell me which one and explain what might be wrong?