409.319A

Aero Thermo Hydro Engineers Nexus Application

Seoul National University
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Problem Set 4

1. Long Pipe **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Water flows from a large reservoir through a very long pipe under constant head h. When the valve is slowly closed, the head h remains constant, but the volume flow rate is reduced.

A. Make a drawing for the water flow and denote the necessary variables to be considered for this problem.
B. Neglecting friction and compressibility of the water, determine the gage pressure just upstream of the valve at any instant during the closure period.
C. Suppose the valve is a short, frictionless nozzle with variable exit area $A_x(t)$. At $t < 0$, prior to valve actuation, a steady flow takes place with $A_x = A$ where A is the cross-sectional area of the pipe. It is desired to program the valve closure such that the volume flow rate decreases linearly in time from its initial steady state value to zero in a period of τ. What relation shall be required for $A_x(t)/A$ to be programmed to perform the above function?

2. Mass Conservation **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A long, flat plate of breadth L which is small compared with the length perpendicular to the sketch is hinged at the left side to a flat wall, and the gap between the plate and wall is filled with an incompressible liquid of density ρ.

A. Draw the plate of breadth L which is small compared with the length perpendicular to the sketch hinged at the left side to a flat wall.
B. If the plate is at a small angle $\theta(t)$ and is depressed at an angular rate $\omega(t) = -d\theta/dt$, obtain an expression for the average liquid speed $u(x,t)$ in the x-direction at station x and time t.

3. Gas Explosion **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Consider a bubble of high pressure gas exploding in an incompressible liquid in a spherically symmetrical fashion. The gas is not soluble in the liquid, and the liquid does not evaporate into the gas. At any instant R is the radius of the bubble, dR/dt is the velocity of the interface, P_g is the gas pressure assumed uniform in the bubble, u is the liquid velocity at the radius r, and P_1 is the liquid pressure at a great distance from the bubble. Do not consider gravity. The following questions pertain to the formulation of an analysis which will lead to the details of the pressure and velocity distributions and to the rate of bubble growth in the limit of inviscid liquid flow.

A. Sketch the bubble of high pressure gas exploding in an incompressible liquid in this problem.
B. Determine the liquid velocity u at the radius r.
C. Obtain the equation describing the rate of growth of the bubble further introducing the surface tension at the gas-liquid interface.
D. What additional information and/or assumptions would be necessary to establish the bubble radius R as a function of time? Explain how you would use this information.

4. Pipe Flow **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A pipe of area A_1 carries a gas at density ρ and velocity V_1. A converging nozzle is mounted at the end of the pipe to increase the gas velocity as it emerges into the atmosphere. The flow in the nozzle is incompressible.

A. Sketch the flow of gas in the pipe with a support
B. Use the momentum theorem to derive the x and y components of force, in excess of those required to support weight, exerted by the nozzle on its support. There is clearly ambiguity in the problem as being stated, since the x component of the force on the support will depend on the compression force applied to the gasket, as well as on the fluid flow. Consider just the flow-induced force which will be exerted when the compression force on the gasket is zero.
C. What gage pressure will the presence of the nozzle induce at the pipe where the area is A_1? You may model the velocity at station (1) as being uniform and assume that the velocity is also uniform at (2).
D. Apart from the assumption that conditions at (2) have attained uniformity, does the result in A depend in any way on the contour of the nozzle between (1) and (2)?
E. What is the direction of the force if $A_2 < A_1$? What if $A_2 > A_1$?

5. Liquid Emulsion **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. There is a liquid emulsion, or a finely-divided mixture of two liquids, of mean density ρ_1 entering a reaction zone of a constant-area reactor with speed V_1. The components of the emulsion react chemically, and leave the reaction zone as a liquid at the density ρ_2. Pitot tubes are installed upstream of the reaction zone. Pressure inside a pitot tube is stagnated at $P_0 = P + \rho V^2/2$. The flow is inviscid, steady and one dimensional, the original emulsion is incompressible, and the liquid leaving the reaction zone is incompressible. Calculate the value of $(P_{0,1} - P_{0,2})/(\rho_1 V_1^{\ 2}/2)$ in terms of the density ratio ρ_2/ρ_1.

6. Jet Pump **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. The device connected between compartments A and B is a jet pump. A jet pump is a simplified device which uses a small, very high-speed jet with relatively low volume flow rate to move fluid at much larger volume flow rates against a pressure differential ΔP. The pump consists of a contoured inlet section leading to a pipe segment of constant area A_2. A small, fast jet of speed V_j and area A_j injects fluid, drawn from compartment *A*, at the entrance plane (1) of the pipe segment. Between (1) and (2), the jet, or the primary stream, and the secondary fluid flow which is drawn in from compartment A via the contoured inlet section mix in a viscous, turbulent fashion and eventually, at station (2), emerge as an essentially uniform-velocity stream. Assume that the flows are incompressible, that the flow from compartment A to station (1) is inviscid, and that, although viscous forces dominate the mixing process between (1) and (2), the shear force exerted on the walls between those stations is small. The pump operates in steady state. Neglect gravity.

A. Derive an expression for ΔP as a function of the total volume flow rate Q from compartment A to compartment B given A_j, A_2, ρ, and V_j. Assume $A_j \ll A_2$ to simplify your expression.
B. Sketch the relationship ΔP vs. Q (the "pump curve") for positive ΔP and Q. Indicate the value of Q when ΔP = 0 (the "short-circuit" volume flow rate). Show that for $A_j \ll A_2$, the latter is large compared with the volume flow rate V_jA_j of the jet.
C. Sketch the pressure distribution along the line a–b for the case when $\Delta P = 0$ and for a case when $\Delta P > 0$.
D. Is your formulation in (a) valid when $Q = 0$, i.e. when the total flow rate for A to B is zero? Explain. What is the minimum volume flow rate Q for which your formulation is valid?

7. Rocket Science **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. After its second booster has been fired, a space vehicle finds itself outside the earth's atmosphere, moving vertically upward at a speed V_0 against gravity g. Its total mass at that point is M_0.At $t = 0$, the vehicle's third stage is turned on and the rocket burns propellant at a mass rate m_r kg/s, ejecting gas from the exit plane (area A_x) at speed V_x relative to the rocket. If the gravitational acceleration remains essentially constant at the vehicle during the rocket firing, determine the velocity $V(t)$ of the vehicle after time t given $M(t)$ the mass of the system at time t. Assume that although the pressure of the gas at the rocket exit plane is P_x (the rocket exhaust is supersonic, and hence the pressure at the exit is not balanced with the zero pressure of space), the effect of the finite exit plane pressure on the thrust is negligible.

8. Lawn Sprinkler **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. There is a lawn sprinkler with two horizontal arms of radial length R, at the termination of which are nozzles with exit area A_2 pointing in a direction which is at an angle θ relative to the tangent of a circumferential line. The sprinkler is free to rotate, but the bearing on which it is mounted exerts a torque $k\omega$ in the direction opposing the rotation, ω being the angular rate of rotation. A constant volume flow rate Q passes through the sprinkler, the flow being incompressible at density ρ.

A. Sketch the lawn sprinkler along with all the parameters involved.
B. Find an expression for the steady-state angular velocity ω of sprinkler in terms of the given quantities R, A_2, θ, Q, ρ, and k.
C. In the steady state, what is the velocity vector of the fluid emerging from the nozzles, as seen by an observer in the non-rotating reference frame? What is the fluid velocity at the nozzle vent if the bearing is frictionless, viz. $k = 0$?
D. If the pipe area at station 1 near the bearing is A_1, and the flow from that point to the nozzles is inviscid, what gage pressure is required at station 1 to sustain the flow rate in this steady state?

9. Sink Flow **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A steady sink flow is set up by injecting water tangentially through a narrow channel near the periphery and letting it drain through a hole at the center. The vessel has a radius R. At the point of injection, the water has a velocity V and depth h_0; the width of the injection channel b is small compared with R. Consider the region of the flow act too close to the drain, and assume that everywhere in the region (i) the flow is essentially incompressible and inviscid, (ii) the radial velocity component $|v_r|$ is small compared with the circumferential velocity component v_θ, and at the periphery.

A. Applying the angular momentum theorem to a control volume for the water between r and R, calculate v_θ.
B. Show that the assumption $|v_r| \ll v_\theta$ is satisfied if $b \ll R$.

10. Circular Tank **10%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. At $t = 0$, a circular tank of radius R contains water at rest, with a depth h. Between $0 < t < \tau$, a water hose is sprayed onto the surface of the water in the tank at a volume flow rate Q and an exit velocity V_j. The jet impacts tangentially on the water at a radius R_j, with an angle θ relative to the horizontal. After the time τ, the hose is turned off. Eventually, all the water in tank will end up rotating like a solid body. Derive an expression for the final angular rate of rotation Ω of the water, assuming shear forces between the water and the walls of the tank are negligible.