409.319A

Aero Thermo Hydro Engineers Nexus Application

Seoul National University
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Problem Set 5

1. Newton's Law **20%**

Newton's law implies that the stress has a more profound attribute, which leads to the concept of the stress tensor. The stress at a given point depends on the orientation of the surface element on which it acts. Take as reference stresses, at a given point $\boldsymbol{R}$ and instant t, the values of the stresses that are exerted on a surface oriented in the positive x-direction, a surface oriented in the positive y-direction, and a surface oriented in the positive z-direction.

A. Sketch the reference stresses at a point in the continuum.
B. Write these three reference stresses, which of course are vectors, in terms of their components.
C. Discuss the surface stress, the stress tensor and its symmetry.
D. Derive the Navier-Stokes equation and its applicable initial and boundary conditions in rectangular coordinates for incompressible flow.
E. Derive the Navier-Stokes equation and its applicable initial and boundary conditions in cylindrical coordinates for incompressible flow.
F. Provide thermophysical properties of selected fluids at 293 K and 1bar including, but not necessarily limited to, CO_2, He, air, water, $C_3H_8O_3$ (glycerin), and Hg. The properties must include density, viscosity, thermal expansion coefficient, bulk modulus, specific heat at constant pressure, and specific heat at constant volume. State clearly where you have taken the data from.

2. Oil in Cylinder **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Oil is confined in a 10 cm diameter cylinder by a piston with a clearance of 0.0002 cm. The piston is 5 cm long, and the oil has a viscosity coefficient of 0.05 kg/ms and a density of 920 kg/m^3. A total weight of 100 kg is applied to the piston.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Estimate the leakage rate of oil past the piston in liters/day.
C. Justify any approximations you may have used in arriving at your estimate.

3. Oil Leak **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. An oil barge has developed a fine crack in its side, running a length L perpendicular to the sketch. Oil leaks out of the crack and runs up the side of the barge inclined at an angle θ in a very thin layer. Assume that the flow in the oil layer is highly viscous, that the oil is less dense than the water ($\rho_o < \rho_w$), and that it is much more viscous than water ($\mu_o \gg \mu_w$).

A. Sketch the problem at hand depicting all the parameters to be considered.
B. If the oil layer is found to have a thickness b, what is the oil volume flow rate Q_{out} through the slit?
C. Describe qualitatively how the field differs when the viscosity of the water is not negligible compared with the oil viscosity.

4. Inclined Rigid Plane **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A rigid plane surface is inclined at an angle θ relative to the horizontal and wetted by a thin layer of highly viscous liquid which begins to flow down the incline.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. If the flow is two-dimensional and in the inertia-free limit, and if the angle of the inclination is not too small, derive the partial differential equation the local thickness $h(x, t)$ of the liquid layer should obey.
C. Demonstrate that the result of B implies that in a region where h decreases in the flow direction, the angle of the free surface relative to the inclined plane will steepen as the fluid flows down the incline, while in a region where h increases in the flow direction, the reverse is true.
D. Does this explain something about what happens to slow-drying paint when it is applied to an inclined surface?
E. Considering the result of C above, do you think that the steady-state solutions of the previous problems would ever apply in practice? Discuss.

5. Annular Flow **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Consider a steady, fully developed laminar flow in an annulus with inside radius R_2 and outside radius R_1.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Find a relation between the pressure gradient dp/dx, the volume flow rate Q, the fluid viscosity μ, R_1, and R_2.
C. Find the limiting form of the relation for a very thin annulus by expressing it in terms of R_1 and $(R_1 - R_2)/R_1$, and taking the limit $(R_1 - R_2)/R_1 \rightarrow 0$.
D. Derive the formula for fully developed laminar flow between parallel flat plates separated by a distance $R_1 - R_2$.
E. In the opposite limit $R_2/R_1 \rightarrow 0$, does the relation of A reduce to the formula for Hagen-Poiseuille flow in a circular pipe of radius R_1? Discuss your answer.

6. Porous Surface **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Consider a flat porous surface with no lengthwise pressure gradient to which suction is applied for the purpose of sucking off the laminar boundary layer. Let us denote by v0 the downward component of velocity at the surface of the plate (y = 0). Then it may be shown that for very small values of v_0/U_∞ compared with unity, the laminar boundary layer becomes constant in both thickness and velocity at large distances from the leading edge, provided that v_0 is constant. We are interested in this case of small v_0/U_∞ and large distance from the leading edge starting with the differential equations of the laminar boundary layer.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Find the velocity profile, by relating u/ U_∞ to $v_0 y/\nu$.
C. Find the displacement thickness Reynolds number, $\delta v_0/\nu$.
D. Find the skin-friction coefficient, $2\tau_0/\rho U_\infty^2$, in terms of v_0, U_∞, and ν.
E. Develop an estimate for the distance in x at which the boundary layer thickness approaches the asymptotic constant value found in Parts A-C.

7. Flat Plate **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A flat plate of breath L and length much greater than its breadth is attached to a plane floor by a hinge. The hinge has a radius R. The plate is initially at a small angle Θ_0 relative to the floor, and the region between it and the floor is filled with a viscous liquid. Starting at t = 0, the plate is forced toward the floor at a constant angular rate of $-d\Theta/dt = \omega$.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Obtain an expression for the pressure distribution $P(x,t)$ under the plate in the limit of highly viscous (inertia-free) flow. The given quantities are L, R, Θ, ω, ρ, μ, and the atmospheric pressure P_a outside the plate.
C. Derive an expression for the vertical force as a function of time which must be applied at the right-hand tip of the plate to make it close down at the specific constant angular rate.
D. Write down the criteria which must be satisfied for your solutions to apply.

8. Circular Bearing Pad **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Consider a circular bearing pad which rests on a flat base through the intermediary of a film of viscous liquid of instantaneous thickness $h(t)$. The load W causes the pad to sink slowly at the speed S, and this squeezes the liquid out from under the pad. Assume that $h \ll D$, that the viscosity is very high, and that the speed S is very small. When h_0 is very small, the time t_∞ is very large. This is the basis for the phenomenon of viscous adhesion, e.g., adhesives such as Scotch tape, or the apparent adhesion of accurately-ground metal surfaces.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Making approximations (state them precisely) consistent with these assumptions, determine the settling speed.
C. An apparatus with two very flat plates of 0.3 m diameter carries a load of 100 kg on a film 0.003 cm thick. If the liquid is an oil with a kinematic viscosity of 10 cm^2/s and a density of 0.93 g/cm^3, estimate the speed S.
D. If the load W is constant, and the gap width is h_0 at time zero, calculate the width $h(t)$ varying with time.
E. Calculate, for the initial conditions of part B, the time (in hours) required for the gap width to be decreased to half its initial value.
F. Suppose now that the initial thickness is h_0, and that a constant upward force F pulls the disk away from the base. Show that the disk will be pulled away in a time $t_\infty = 3\pi\mu D^4/64 h_0^2 F$.

9. Cylindrical Container **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. An infinitely long, cylindrical container of radius R rotates at the angular speed Ω. It contains water which is also in solid body rotation with angular speed Ω. At time t = 0, the container suddenly stops rotating, and the contained water gradually comes to rest. In all that follows, ignore the possible effects of turbulence and other instabilities.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Sketch curves of V_Θ vs. r, showing how the circumferential velocity varies with radius for several successive times $t > 0$.
C. What is the order of magnitude of the time, t_R, up to which the Rayleigh's solution for impulsive start of a flat plate would describe the motion near the wall?
D. Given that Ω = 33 rpm, R = 10 cm, and that the fluid is water at 20°C, make a very rough estimate of the time t in seconds required for most of the motion to disappear.

10. Gas Bubble **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Consider a gas bubble of fixed mass and radius $R(t)$ which is expanding or contracting in an infinite sea of incompressible liquid. The speed of the interface is dR/dt. The local Eulerian coordinate in the liquid is r. Let P_R, P, and P_∞ be, respectively the pressure at $r = R$ (on the liquid side of the interface), at $r = r$, and at $r = \infty$. Choose a control volume taking the shape of a hollow sphere with inner control surface at radius $R(t)$, which moves outward.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Determine the viscous contribution to the normal stress τ_{rr} in the liquid.
C. Show that the dimensionless overpressure $(P_R - P_\infty)/\rho(dR/dt)^2$ is independent of the fluid's viscosity.