409.319A

Aero Thermo Hydro Engineers Nexus Application

Seoul National University
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Problem Set 6

1. Turbulent Flow in Pipe **20%**

This note is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. The notion of being steady in time in laminar flow is slightly different when you are dealing with turbulent flows. The important fact is that when you try to deduce the governing equations of motion you should carry these fluctuation terms all along. Finally, you will end up getting an equation very similar to Navier-Stokes which is called the Reynolds equation. There is still an ongoing debate on whether the idea of time averaging in the turbulent flow is the best approach but those discussions are way beyond the scope of this course.

A. Sketch a laminar flow (steady vs. unsteady) in a pipe.
B. Sketch a turbulent flow (steady vs. unsteady) in a pipe.
C. In turbulent flow your velocity is generally not steady and you have fluctuations. What is going to be your approach to write all the parameters in the conservation equations?
D. Derive the Reynolds equation, and explain what each term signifies physically and mathematically.
E. For the simple pipe flow, in the turbulent case unlike the laminar case), even though the flow is "steady and fully developed in mean, the left hand side will not turn into zero. What is the form of the equation when it is simplified?

2. Metal Ball **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A metal ball falls at steady speed in a large tank containing a viscous liquid. The ball falls so slowly that it is known that the inertia forces may be ignored in the equation of motion compared with the viscous forces.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Perform a dimensional analysis of this problem, with the aim of relating the speed of fall V, to the diameter of the ball D, the mass density of the ball ρ_b, the mass density of the liquid ρ_l, and any other variables which play a role. Note that the effective weight of the ball is proportional to $(\rho_b - \rho_l)g$.
C. Suppose that an iron ball (specific gravity = 7.9, D = 0.3 cm) falls through a certain viscous liquid (specific gravity = 1.5) at a certain steady-state speed. What would be the diameter of an aluminum ball (specific gravity = 2.7) which would fall through the same liquid at the same speed assuming inertial forces are negligible in both flows?

3. Shock Wave **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A strong explosion like an atomic bomb causes a spherically symmetric shock wave to move through the air radially out from the origin. As the shock sweeps by, it causes a sudden rise in the pressure and sets the initially static air into radially outward motion. It can be argued from strong shock wave theory that if the undisturbed atmosphere is homogeneous at a density ρ_a, the velocity vs of the shock, as well as the pressure p_s and the wind speed just behind the shock wave, should depend only on the density ρ_a, the total distance r_s of the shock wave from the origin, and the total energy E released by the explosion.

A. Sketch the problem at hand depicting all the parameters to be considered.
B. Derive the relations for the speed of the shock wave v_s and p_s.
C. Obtain an expression for the shock's radial position as a function of time. The expression may involve one unknown dimensionless constant. Show how the strengths of two different bomb explosions, as measured by their energy releases, can be compared based on film information about their shock wave positions as a function of time.

4. Incompressible Flow **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. Consider an incompressible flow through a series of geometrically similar machines such as fans, pumps, hydraulic turbines, etc. In what follows, Q denotes volumetric flow, ω rotational speed, D impeller diameter, μ fluid viscosity, and ρ fluid density.

A. Show that dynamic similarity requires that $Q/\omega D^3$ and $\rho Q/\mu D$ be fixed.
B. Show that if $Q/\omega D^3$ and $\rho Q/\mu D$ are fixed in a series of tests, then $\Delta p/\rho\omega^2 D^2$ must remain constant, where Δp is the change in head across the machine, expressed in pressure units.
C. Find the form of the relation between the work output per unit mass of fluid W, and the given variables, in a series of tests where $Q/\omega D^3$ and $\rho Q/\mu D$ are fixed.

5. Pressure Gradient **20%**

This problem is from "Advanced Fluid Mechanics Problems" by A.H. Shapiro and A.A. Sonin. A fluid of density ρ and viscosity μ flows through a long pipe of diameter D at the volume flow rate Q.

A. If the flow is laminar (i.e., totally steady) and fully developed (the velocity profile and the pressure gradient no longer change with downstream distance x), what is the pressure gradient in the direction of flow?

B. If the flow is turbulent (i.e., unsteady) but steady and fully developed in the mean, what is the mean pressure gradient in the direction of flow?

6. Fish Business **20%**

This problem is from “Advanced Fluid Mechanics Problems” by A.H. Shapiro and A.A. Sonin. You are concerned with the mechanics of fish propulsion. To determine how the thrust force generated by a fish of a given geometry depends on fish size (L = fish length) and on the frequency of oscillation of the tail (f cycles/sec), you build a mechanical model, having this geometry, of length L = 1 m. You then mount this model in a fixed position deep within a large tank containing stagnant water at room temperature, and measure the thrust force F, over a large range of frequency of tail oscillation.

A. You find that your data can be described by an empirical equation. What is it?
B. Now you want to infer, from these results, the thrust generated by fish of other sizes held in still water having different temperature (i.e. different density, viscosity). What relation must be satisfied between the frequency, size, and fluid condition of the real fish and of the model experiments?
C. From the empirical equation given above for the thrust of a 1 m model in room temperature water, develop a formula for the thrust of a fish of any given size and tail frequency, held in water at any given density and viscosity.

7. G.I. Taylor and the Trinity Explosion **20%**

This problem is from “Math Methods,” D. Craig, 2007–01–19. During World War II, the British government cooperated with the U.S. on the development of the atomic bomb in the Manhattan project. G.I. Taylor, a British fluid dynamicist, was asked by his government to study mechanical ways of measuring the bomb’s yield, or energy output. Taylor was not directly involved in the bomb’s development, and for security reasons worked independent of the U.S. project. He knew that the energy would be released from a small volume, and would produce a very strong shock wave that would expand in approximately a spherical shape. He used dimensional analysis to estimate how the radius would scale with the other physical variables. From his work in fluids, Taylor assumed the relevant variables would be r the radius of the shock front, ρ density of surrounding air, E energy released by the device, t the time at which the front reaches r. In 1947 a movie of the Trinity test explosion was released to the public. In one frame r = 100 m at a time of t = 0.016 s after the explosion. $\rho \approx 1.1$ kg/m^3 at that altitude according to. http://nuclearweaponarchive.org/Usa/Tests/Trinity.html.

A. Knowing that 1000 tons of TNT (a kiloton) releases about 4.2×10^{12} J, determine the energy released from the Trinity explosion in terms of the TNT equivalent. Compare your result against the actual yield of 18–22 kilotons.
B. Remember mathematical functions only take dimensionless arguments. This is shown by power series expansions. In this case the leading term is obviously dimensionless, and all terms added to it must be also. In general, a function has terms of many different orders, which must be dimensionless to add up. Some ratios of variables and their derivatives can lead to ambiguous cases. Give an example for this.
C. Derivatives and ratios are indistinguishable to a dimensional analysis. Give an example.
D. Dimensional analysis is an aid to insight: it cannot completely describe the physics. Your thoughts?

8. Touching Down on Mars Safe & Sound **20%**

You may as well be fascinated by the video clip https://ocw.mit.edu/resources/res-tll-004-stem-concept-videos-fall-2013/videos/problem-solving/dimensional-analysis/. This video leads you through the problem solving method of dimensional analysis. You use dimensional analysis to determine the diameter of a parachute needed to slow a rover to 90 m/s in order to safely land on Mars. You’ll see how the general formula found using dimensional analysis applies on any planet, allowing for the use of experimental data collected on Earth to Mars. After watching this video, you will be able to use dimensional analysis to estimate the size of a parachute canopy that can slow a rover to 90 m/s on its descent to Mars. (MIT © 2012) Watch the movie and summarize the lessons learned.

9. Submarine Running Time **20%**

Submarines are powered by energy sources that do not require air. Excluding nuclear power, batteries have been the most commonly used energy source. Unfortunately, batteries have a low energy density and tend to be big and heavy. Because underwater propulsion is all about energy, the key figure of merit for an underwater propulsion system is its total available energy. Watch http://mathscinotes.com/2012/04/dimensional-analysis-and-submarine-running-time/ and summarize the dimensional analysis you’ve learned from this practice.

10. Coastal Hydrology **20%**

Surface-water modeling is an important tool to support many types of hydrologic investigations. As is generally the case in modeling, it allows the prediction of future conditions and investigation of causative factors. Surface water models offer a variety of formulas to represent flow and water level. Formulas that make simplifying assumptions by neglecting certain terms provide for solutions requiring fewer calculations and may require less parameterization. Hydrodynamic flow governing equations account for spatial and temporal changes in momentum, whereas simpler formulations retain only the frictional and gravitational forcing terms or combine an empirical flow relationship with mass conservation. Read through https://file.scirp.org/pdf/CWEEE_2014042515355921.pdf. Its authors present a dimensional analysis of the magnitude of the temporal and inertial terms in the surface-water flow equations and determine the conditions under which these inertial terms have sufficient magnitude to be required in computations. What was your takeaway? Summarize the lessons that you have learned from the paper as quantitatively as you can.