Statement

Air enters the compressor of an ideal air–standard Brayton cycle at 100 kPa, 300 K, with a volumetric flow rate of 5 m^3/s.

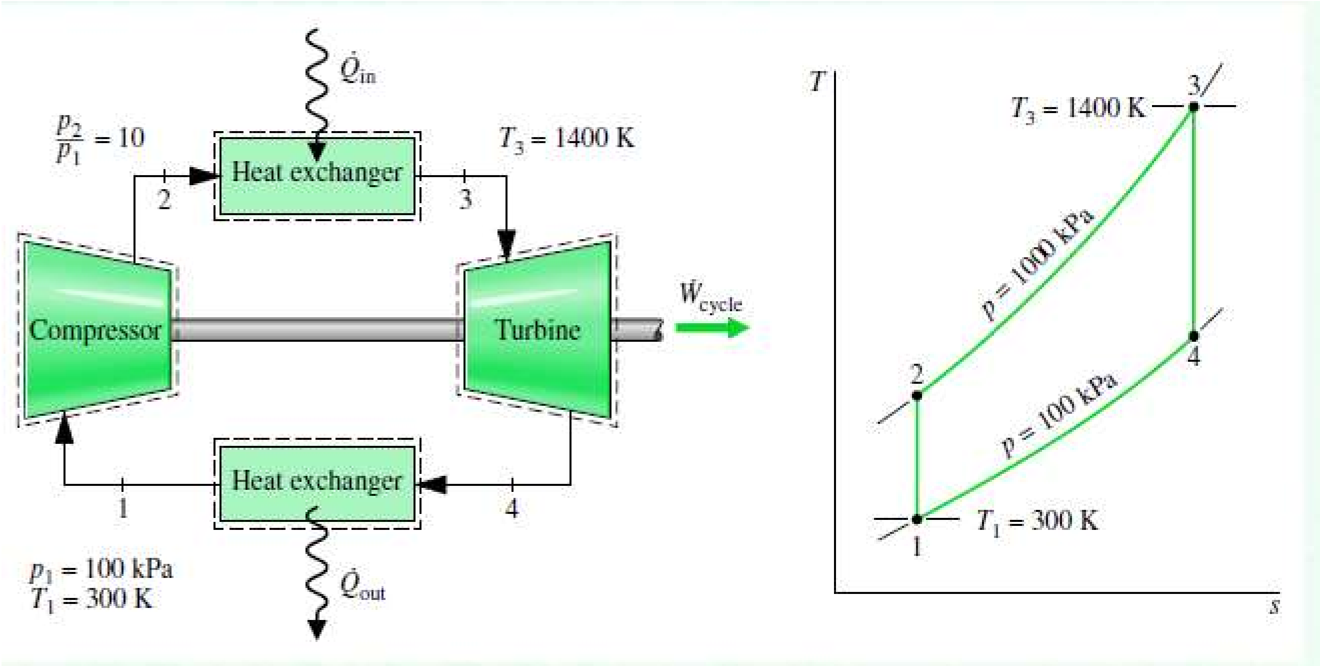
The compressor pressure ratio is 10. The turbine inlet temperature is 1400 K. Determine

(a) the thermal efficiency of the cycle,

(b) the back work ratio,

(c) the net power developed, in kW.

Figure


$\dot{Q}_{in}$
$\frac{p_2}{p_1} = 10$
Heat exchanger
$T_3 = 1400$ K
2
3
Compressor
Turbine
$\dot{W}_{cycle}$
Heat exchanger
1
4
$p_1 = 100$ kPa
$T_1 = 300$ K
$\dot{Q}_{out}$
T
$T_3 = 1400$ K
3
$p = 1000$ kPa
2
4
$p = 100$ kPa
$T_1 = 300$ K
1
s

Solution

- Part (a) for thermal efficiency

for enthalpy state 1 use table(ideal gas properties of air)
at T_1=300K, h_1=300.19kJ/Kg, p_{r1}=1.386

$p_{r2}=(p_2/p_1)p_{r1}$

p_{r2}=10*1.386 =13.86

By interpolating the table h_2=579.9kJ/Kg

Similarly for specific enthalpy at state 3 use table at
T_3=1400K,h_3=1515.4kJ/Kg, p_{r3}=450.5

$P_{r4}=(p_3/p_4)p_{r3}$=450.5*1/10=45.05

h_4=808.5kJ/Kg

$\eta=(W_t/m-W_c/m)/(Q_{in}/m)$

$\eta=[(h_3-h_4)-(h_2-h_1)]/(h_3-h_2)$

$\eta=(706.9-279.7)/935.5=0.457$

- Part(b) Back work ratio

$bwr=(W_c/m)/(W_t/m)$

$bwr=(h_2-h_1)/(h_3-h_4)$

$bwr=279.7/706.9$

$bwr=0.396$

- Part(c) Net power developed

W_{cycle}=m[(h_3-h_4)-(h_2-h_1)

W_{cycle}=5.807(706.9-279.7)=2481kW

$$T\,ds = dh - v\,dp$$

$$ds = \frac{dh}{T} - \frac{v}{T}dp$$

$dh = c_p(T)\,dT$, and $pv = RT$.

$$ds = c_p(T)\frac{dT}{T} - R\frac{dp}{p}$$

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R\ln\frac{p_2}{p_1} \qquad (1)$$

The value of specific entropy is set to zero at the state T=0K and P=1atm,
and the entropy at a state where temperature is T and P=1 atm relative to reference state and value is

$$s^\circ(T) = \int_0^T \frac{c_p(T)}{T}dT$$

$$\int_{T_1}^{T_2} c_p\frac{dT}{T} = \int_0^{T_2} c_p\frac{dT}{T} - \int_0^{T_1} c_p\frac{dT}{T}$$
$$= s^\circ(T_2) - s^\circ(T_1) \qquad (2)$$

Put equation (2) in (1)

$$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}$$

For same entropy(isentropic state)

$$0 = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}$$

$$s^\circ(T_2) = s^\circ(T_1) + R \ln \frac{p_2}{p_1}$$

$$p_2 = p_1 \exp\left[\frac{s^\circ(T_2) - s^\circ(T_1)}{R} \right]$$

$$\frac{p_2}{p_1} = \frac{\exp[s^\circ(T_2)/R]}{\exp[s^\circ(T_1)/R]}$$

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}}$$