Statement

Derive the relation $c_p = -T(\partial^2 g/\partial T^2)_p$.

Solution

consider $h(T,p)$, $s(T,p)$

$dh= (\delta h/ \delta T)_p dT+(\delta h/ \delta p)_T dp$

$dh=C_p dT+ +(\delta h/ \delta p)_T dp$

Forming the differential of $s(T,p)$

$ds= (\delta s/ \delta T)_p dT+(\delta s/ \delta p)_T dp$

Using equation $dh= Tds+vdp$

$C_p dT+ +(\delta h/ \delta p)_T dp=T[(\delta s/ \delta T)_p dT+(\delta s/ \delta p)_T dp]+vdp$

$[C_p-T(\delta s/ \delta T)_p)]dT=[(\delta s/ \delta p)_T+v-(\delta h/ \delta p]dp$

Consider t, P are independent ,fixed P and vary T, so

$dP=0, dT\neq 0$

$(C_p - T[(\delta s/\ \delta T)_p)dT=0$

$C_p = T\ (\delta s/\ \delta T)_p$

$-s = (\delta g/\ \delta T)_p$

$-(\delta s/\ \delta T)_p = (\delta g^2/\ \delta T^2)_p$

$Cp/T = -(\delta g^2/\ \delta T^2)_p$

$Cp = -T(\delta g^2/\ \delta T^2)_p$