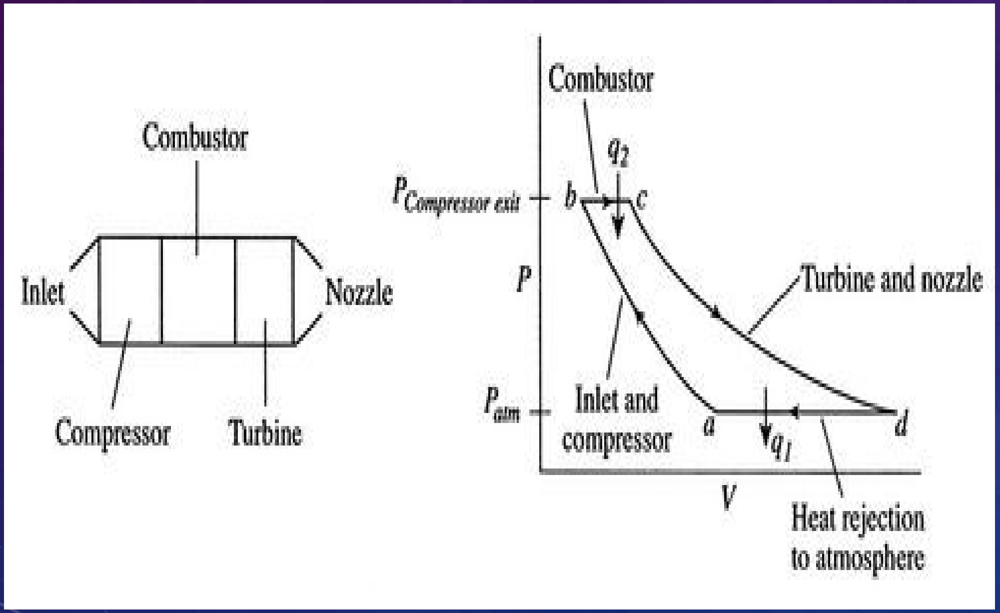
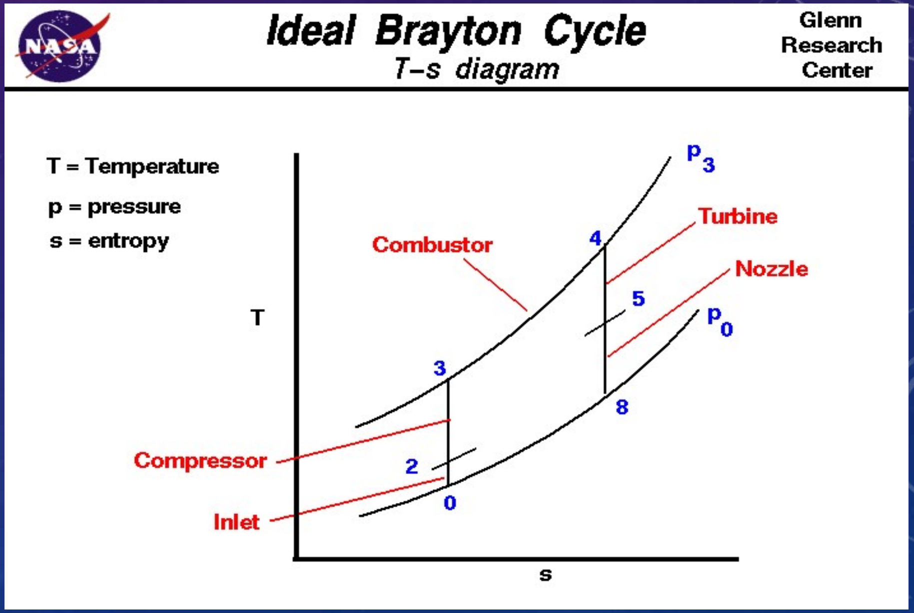
PROBLEM

Find the equation for the efficiency of the Brayton cycle (Gas Power Cycle) and then solve the following question:





SOLUTION

- Write the first law for the cycle and find the relation between work and heat , thus:
- $\delta Q/m = \delta W/m$…1
- Find the expression for the heat added by combustor and heat rejected to atmosphere by referring to the P~V diagram on the previous slide:
- $q_{added} = C_P(T_c - T_b)$, similarly for heat rejected $q_{rejected} = C_P(T_d - T_a)$
- Find the work done according to equation 1
- Find efficiency η= $\frac{Work\ done}{Heat\ added} = 1 - (T_d - T_a)/\ (T_c - T_b)\ or, 1 - T_a(T_d/T_a - 1)/T_b\ (T_c/T_b - 1)$

- T_a is compressor inlet which is the ambient temperature and is known, evaluate T_b and T_d, use the isoentropic assumption for the process and write:

- $ds = 0 = \frac{du+}{T} = \frac{du+pdv-vd}{T} = \frac{dh-v}{T} = \frac{C_p dT}{T} - vdp/T$ *integrating between a and b states we will get,* $C_p \ln\left(\frac{T_a}{T_b}\right) = Rln(\frac{P_a}{P_b})$

- *Again Cp-Cv=R, thus* $Cp = \frac{Rk}{k-1}$ *replacing we get,* $\left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}} = \frac{P_a}{P_b}$ *Also* $\frac{P_c}{P_d} = \left(\frac{T_c}{T_d}\right)^{\frac{k}{k-1}}$ *also* , $\left(\frac{T_a}{T_b}\right)^{\frac{k}{k-1}} = \left(\frac{T_d}{T_c}\right)^{\frac{k}{k-1}}$ *(why ?)*

- *Or* $\frac{T_d}{T_a} = \frac{T_c}{T_b}$ *Replace in the efficiency equation to get* $1 - \left(\frac{T_a}{T_b}\right) = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}}$ *(efficiency in terms of compression ratio).*

- Now do the following question:
- In a Brayton cycle based power plant, the air at the inlet is at 27 degree C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800 degree C. Find (a) the compressor work per kg of air (b) the turbine work per kg or air (c) the heat supplied per kg of air, and (d) the cycle efficiency.

SOLUTION

- For compressor: $(h - vdp) = C_p dT - vdp = 0 \Rightarrow W = C_p dT$, to evaluate dT we need input and exit temperature, inlet is known (ambient), to evaluate outlet temperature use the equation developed previously i.e. $\left(\frac{T_b}{T_a}\right)^{\frac{k}{k-1}} = \frac{P_b}{P_a}$, pressure ratio is given as 6.25, k for air is 1.4, thus plugging everything in the equation gives T_b=506.69 K
- Wcomp=1.005*(506.69-300)=207.72 kJ/kg
- Similarly for turbine 6.25 is pressure ratio, k for air is 1.4 and inlet temperature is 800 degree celcius thus exit temperature is 635.29 K
- Wturbine=1.005 (1073-635.29)=439.89 kJ/kg
- Similarly heat added in the constant pressure process is (combustor)=1.005 (1073-506.69)=569.14kJ/kg
- Thus efficiency is (439.89-207.72)/569.14=0.408