Engineering Economic Analysis

Spring 2019

Problem set 1 - Solution

1. *Text 1.7*

- 1^{st} Question: He will set a price of 25 and rent 50 apartments.
- 2^{nd} Question: He will set a price of 30 and rent 40 apartments.

Text 2.6

$$(p_1 + t)x_1 + (p_2 - s)x_2 = m - u$$

Text 4.5

1^{st} Question: Quasi-linear preferences.

2^{nd} Question: Yes.

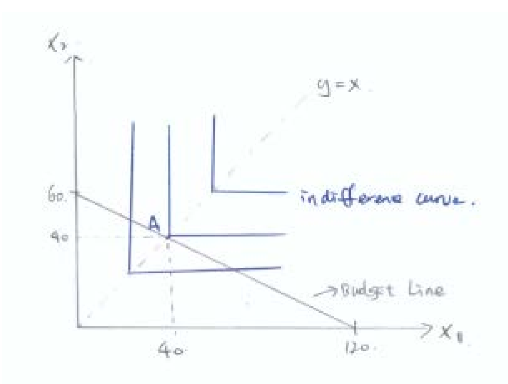
2. (A) E, who is willing to pay $10 for an apartment, will sublet to F, who is willing to pay $18.

(B) $18

(C) A, B, C, D, F.

(D) It’s the same.

3. (A)



(B) $10x_1 + 20x_2 = 1200$

(C) $x_1 = x_2$

(D) (40, 40)

(E) 400 minutes for the first examination, and 800 minutes for the second examination.

4. - Marshallian demand function

$$x_1^* = \begin{cases} \frac{m}{p_1} & \text{if } p_1 < p_2 \\ 0 \text{ or } \frac{m}{p_1} & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}, \quad x_2^* = \begin{cases} 0 & \text{if } p_1 < p_2 \\ 0 \text{ or } \frac{m}{p_2} & \text{if } p_1 = p_2 \\ \frac{m}{p_2} & \text{if } p_1 > p_2 \end{cases}$$

- Indirect utility function

$$v(p_1, p_2, m) = \frac{m}{\min\{p_1, p_2\}}$$

- Expenditure function

$$e(p_1, p_2, u) = \min\{p_1, p_2\}u$$

5. **(**A) By Roy's identity, $x_j^* = -\dfrac{\partial v / \partial p_j}{\partial v / \partial m}$.

Then, we can calculate the demand functions as $x_1^* = x_2^* = \dfrac{m}{p_1 + p_2}$.

(B) $e(p_1, p_2, u) = (p_1 + p_2)u$

(C) $u(x_1, x_2) = \min\{x_1, x_2\}$

6. (A) Quasi-linear preferences

(B) Less than $u(1)$

(C) $v(p_1, p_2, m) = \max\{u(1) - p_1 + m, m\}$

7.

Write the Lagrangian

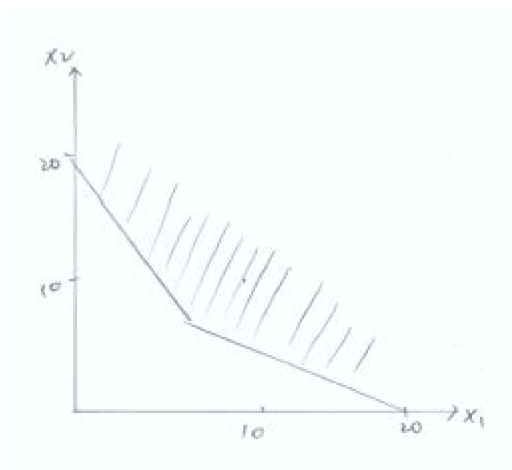
$$L(\mathbf{x},\lambda) = \frac{3}{2}\ln x_1 + \ln x_2 - \lambda(3x_1 + 4x_2 - 100)$$

Now, equating the derivatives with respect to x_1, x_2, and λ to zero, we get three equations in three unknowns

$$\frac{3}{2x_1} = 3\lambda$$
$$\frac{1}{x_2} = 4\lambda$$
$$3x_1 + 4x_2 = 100$$

Solving these equations, we get $\left(x_1^*, x_2^*\right) = (20,10)$.

8. (A)



(B) The slope of a budget line is $-p_1 / p_2$. If the budget line is steeper than 2, $x_1 = 0$. Hence the condition is $p_1 / p_2 > 2$

(C) If the budget line is flatter than 1/2, $x_2 = 0$, so the condition is $p_1 / p_2 < 1/2$

(D) If the optimum is unique and neither x_1 nor x_2 is zero, it must occur where $x_2 - 2x_1 = x_1 - 2x_2$. This implies that $x_1 = x_2$, and it means $x_1 / x_2 = 1$.

9.

Value tax: $p_1 \to p_1(1+t)$

(A) The problem can be formulated as $\begin{array}{ll} \max & \ln x_1 + x_2 \\ s.t. & p_1(1+t)x_1 + x_2 = m \end{array}$

It can be rewritten as $\max \ln x_1 + \left(m - p_1(1+t)x_1\right)$

From the first order condition,

$$\frac{\partial U}{\partial x_1} = \frac{1}{x_1} - p_1(1+t) = 0, \quad \therefore x_1^* = \frac{1}{p_1(1+t)}, x_2^* = m-1$$

Note that when $m \le 1, x_2^* = 0$.

Therefore,

$$\left(x_1^*, x_2^*\right) = \begin{cases} \left(\frac{1}{p_1(1+t)}, m-1\right) & \text{if } m > 1 \\ \left(\frac{m}{p_1(1+t)}, 0\right) & \text{if } m \le 1 \end{cases}$$

(B) Since $m > 1$, indirect utility function can be written as $v(\tilde{p}, m) = \ln\left(\frac{1}{p_1(1+t)}\right) + m - 1$.

- 1st policy: $t \to t - \alpha$, then the indirect utility $v^1(\tilde{p}, m) = \ln\left(\frac{1}{p_1(1+t-\alpha)}\right) + m - 1$.

- 2nd policy: Tax revenue $R = t \times p_1 \times x_1^* = t \times p_1 \times \frac{1}{p_1(1+t)} = \frac{t}{1+t} = S$

  So, the indirect utility function $v^2(\tilde{p}, m) = \ln\left(\frac{1}{p_1(1+t)}\right) + m - 1 + \frac{t}{t+1}$.

v^1 should be bigger than v^2, if the consumer prefer the first policy.

$$\therefore \ln\left(\frac{1}{p_1(1+t-\alpha)}\right) > \frac{t}{1+t} + \ln\left(\frac{1}{p_1(1+t)}\right) \Leftrightarrow \ln\left(\frac{1+t}{1+t-\alpha}\right) > \frac{t}{1+t} \Leftrightarrow \alpha > (1+t)\left(1 - e^{-\frac{t}{1+t}}\right).$$