Solution of Problem Set #2

1.

(a)

By the utility maximization, the demand function for both products can be found

$$x_1 = \frac{\alpha m}{(\alpha+\beta)P_1}, x_2 = \frac{\beta m}{(\alpha+\beta)P_2}$$

Thus the change in surplus from the price change of product 2 is

$$\int_{P_2}^{2P_2} x_2(P,m)\,dP = \int_{P_2}^{2P_2} \frac{\beta m}{(\alpha+\beta)P_2}\,dP = \frac{\beta m}{(\alpha+\beta)}\int_{P_2}^{2P_2} \frac{1}{P}\,dP = \frac{\beta m}{(\alpha+\beta)}\ln 2$$

(b)

By the expenditure minimization, the Hicksian demand function can be found, and afterwards plug them into the objective function. Then the expenditure function is given by

$$e(P,u) = (\alpha+\beta)\left(\frac{P_1}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{P_2}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} u^{\frac{1}{\alpha+\beta}}$$

Indirect utility function at price vector P' is $v(P,m) = \left(\frac{m}{\alpha+\beta}\right)^{\alpha+\beta}\left(\frac{\alpha}{P_1}\right)^{\alpha}\left(\frac{\beta}{2P_2}\right)^{\beta} = u'$

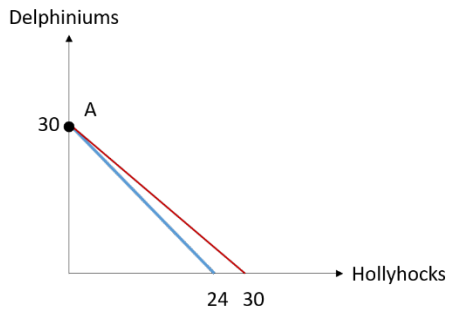
Then this individual would be willing to accept a reduction of income such that

$$e(P',u') - e(P,u') = m - \frac{m}{2^{\frac{\beta}{\alpha+\beta}}} = m\left(1 - \frac{1}{2^{\frac{\beta}{\alpha+\beta}}}\right)$$

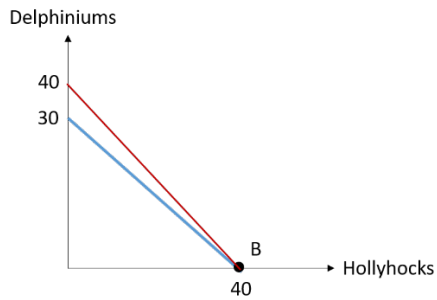
2.

(a) Yes. All due to income effect.

(b)



(c)



(d) $120.

(e) All substitution effects.

3.

Ellsworth's demand functions for x-good and the y-good takes the form $x = y = \frac{150}{p_x + p_y}$. Plugging this into the utility function, we find that the indirect utility function takes the form $v\left(p_x, p_y, 150\right) = \frac{150}{p_x + p_y}$. Hence A is the solution to $\frac{150 - A}{1+1} = \frac{150}{1+2}$, and B is the solution to $\frac{150}{1+1} = \frac{150 + B}{1+2}$. Solving, we have A=50 and B=75.

4.

$$D(p) = A - p + \ln m$$

Price elasticity

$$\varepsilon = \frac{p}{q} \cdot \frac{dq}{dp} = \frac{p}{A - p + \ln m} \cdot (-1)$$

Thus, $|\varepsilon| = \dfrac{p}{A - p + \ln m}$

The condition is $|\varepsilon| < 1$

$$\therefore \frac{p}{A - p + \ln m} < 1$$

$$p < A - p + \ln m$$

$$p < \frac{A + \ln m}{2}$$

5.

(a)

$$\min\ p_1 x_1 + p_2 x_2$$

$$s.t.\ x_1^2 x_2 \geq \bar{u}$$

$$x_2 = \frac{\bar{u}}{x_1^2}$$

$$\max\ p_1 x_1 + p_2 \frac{\bar{u}}{x_1^2}$$

$$F.O.C.\ p_1 - 2\bar{u} p_2 x_1^{-3} = 0$$

$$\therefore x_1 = \left(\frac{p_1}{2\bar{u} p_2}\right)^{-1/3}, x_2 = \left(\bar{u}\right)^{1/3} \left(\frac{p_1}{2 p_2}\right)^{2/3}$$

$$e\left(p_1, p_2, \bar{u}\right) = p_1 \left(\frac{p_1}{2\bar{u} p_2}\right)^{-1/3} + p_2 \left(\bar{u}\right)^{1/3} \left(\frac{p_1}{2 p_2}\right)^{2/3}$$

$$v\left(p_1, p_2, m\right) = \frac{4m^3}{27 p_1^2 p_2}$$

$$x_1 = -\frac{\partial v / \partial p_1}{\partial v / \partial m} = \frac{2m}{3 p_1}$$

$$x_2 = -\frac{\partial v / \partial p_2}{\partial v / \partial m} = \frac{m}{3 p_2}$$

(b)

By total differentiation:

$$\begin{cases} -2\lambda x_2 dx_1 - 2\lambda x_1 dx_2 - 2x_1 x_2 d\lambda = -dp_1 \\ -2\lambda x_1 dx_1 - x_1^2 d\lambda = -dp_2 \\ -2x_1 x_2 dx_1 - x_1^2 dx_2 = -d\bar{u} \end{cases}$$

$$\begin{bmatrix} -2\lambda x_2 & -2\lambda x_1 & -2x_1 x_2 \\ -2\lambda x_1 & 0 & -x_1^2 \\ -2x_1 x_2 & -x_1^2 & 0 \end{bmatrix} \begin{bmatrix} dx_1 / d\bar{u} \\ dx_2 / d\bar{u} \\ d\lambda / d\bar{u} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

By Cramel's rule:

$$\frac{dx_2}{d\bar{u}} = \frac{\begin{vmatrix} -2\lambda x_2 & 0 & -2x_1 x_2 \\ -2\lambda x_1 & 0 & -x_1^2 \\ -2x_1 x_2 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -2\lambda x_2 & -2\lambda x_1 & -2x_1 x_2 \\ -2\lambda x_1 & 0 & -x_1^2 \\ -2x_1 x_2 & -x_1^2 & 0 \end{vmatrix}} = \frac{2\lambda x_1^2 x_2}{6\lambda x_1^4 x_2} = \frac{1}{3x_1^2} > 0$$

Thus, disapprove!

6.

(a)

Hicksian demand function

$$h_x(\tilde{p}, u) = \frac{\partial e}{\partial p_x} = \frac{1}{2}\sqrt{\frac{p_y}{p_x} u}$$

$$h_y(\tilde{p}, u) = \frac{\partial e}{\partial p_y} = \frac{1}{2}\sqrt{\frac{p_x}{p_y} u}$$

Indirect utility function

$$x(\tilde{p}, m) = -\frac{\partial v / \partial p_x}{\partial v / \partial m} = -\frac{-m^2 / p_x^2 p_y}{2m / p_x p_y} = \frac{m}{2p_x}$$

$$y(\tilde{p}, m) = \frac{m}{2p_y}$$

(b)

Indirect utility(optimal utility) when

$p_x = 1, p_y = 1, m = 100$

$$v(\tilde{p}, m) = \frac{m^2}{p_x p_y} = 10000$$

When $p_x = 1.1, p_y = 1, m = 10,000$

$$v(\tilde{p}', m) = \frac{(m')^2}{1.1} = 10,000$$

$$\therefore m' = \sqrt{11,000} \simeq 104.88$$

$$\therefore 104.88 - 100 = \$4.8$$

(c)

Under $p_x = 1, p_y = 1, m = 100$

$$x(\tilde{p}, m) = y(\tilde{p}, m) = \frac{100}{2 \times 1} = 50$$

With $p_x = 1.1, p_y = 1,$ to buy $x = y = 50,$ the consumer need the income

$$m' = 1.1 \times 50 + 1 \times 50 = 105$$

Thus new environment for the consumer is $p_x = 1.1, p_y = 1, m = 105$

Thus, $x(\tilde{p}, m) = \frac{105}{2.2} = 47.7$

$$y(\tilde{p}, m) = \frac{105}{2} = 52.5$$