Engineering Economic Analysis

Spring 2019

Problem set 4 Solution

Due: 2019.06.18(Tue)

1.

(a)

The average cost curve is $\frac{c(\mathbf{w}, y)}{y} = \frac{y^2+1}{y} w_1 + \frac{y^2+2}{y} w_2$

Since it is convex, it has a unique minimum at $y_m = \sqrt{\frac{w_1 / w_2 + 2}{w_1 / w_2 + 1}}$.

The derivative of y_m with respect to w_1 / w_2 is negative, so the minimum of the average cost shifts to the left (right) as w_1 / w_2 increases (decreases).

(b)

$$\frac{\partial c(\mathbf{w}, y)}{\partial y} = 2y(w_1 + w_2)$$

$$\therefore y(p) = \frac{p}{2(w_1 + w_2)}$$

(c)

$$Y(p) = \begin{cases} \text{arbitrarily large amount} & \text{if } p > 2y_m(w_1 + w_2) \\ 0 & \text{otherwise} \end{cases}$$

2.

(a) For the competitive firms, their profit-maximizing output is $MC = y = p$ for each firm.

(b) For the monopolist, the profit-maximizing problem is $\max(1000 - 50p - 50p)\,p$

Thus, $p = 5$ and $y_m = 500$.

(c) $y_c = 5 \times 50 = 250$

3.

(a) Price equals to marginal cost: $p = y$, $Y = 2p$.

(b) Set demand equal to supply $90 - p = 2p$, then $p^* = 30 \text{ and } Y^* = 60$.

(c) Let p be the price paid by consumers. Then the domestic firms receive a price of p and the foreign firms receive a price of $p - 3$. Demand equals supply gives us $90 - p = p + (p - 30)$ $\therefore p^* = 31$

(d) The supply of umbrellas by domestic firms is 31 and by foreign firms is 28.

4.
The revenue function is

$$R(p) = \begin{cases} 10 \; if \; p \le 20 \\ 0 \; if \; p > 20 \end{cases}$$

Since the more products the firm produces, the less is the profit, the monopolist will want to produce the smallest possible output. This will happen when $p = 20$ and $y = 1/2$

5.
Under the ad valorem tax, we have $(1-\tau)P_D\left(1+\frac{1}{\varepsilon}\right) = c$.

Under the output tax, we have $P_D\left(1+\frac{1}{\varepsilon}\right) = c + t$.

Solve each equation for P_D, set the results equals to each other, and solve t to find $t = \frac{c\tau}{1-\tau}$

6.
(a)
In market 1, the profit is $(a_1 - b_1 p_1)p_1$. Thus, the profit-maximizing choice of p_1 is $p_1 = \frac{a_1}{2b_1}$.

Similarly, the profit-maximizing choice of p_2 is $p_2 = \frac{a_2}{2b_2}$.

These will be equal when $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

(b)
In market i, the profit is $(A_i p_i^{-b_i})(p_i - c)$ for i=1, 2.
F.O.C. gives $-b_i A_i p_i^{-b_i-1}(p_i - c) + A_i p_i^{-b_i} = (1-b_i)A_i p_i^{-b_i} - b_i c A_i p_i^{-b_i-1} = 0$
$\therefore (1-b_i)p_i - b_i c = 0$
$\therefore c = \frac{1-b_i}{b_i} p_i$
That is, $c = \frac{1-b_1}{b_1} p_1 = \frac{1-b_2}{b_2} p_2$.
Thus, $p_1 = p_2$ if and only if $b_1 = b_2$.

7.
(a) $MC = p = y$
(b) $y = 50p$
(c) $D_m(p) = 1000 - 100p$

(d) $y_m = 500$

(e) $p = 5$

(f) $y_c = 50 \times 5 = 250$

(g) $Y = y_m + y_c = 750$

8.

(a)

Follower(Firm 2)'s problem

$$\max\left(a - q_1 - q_2\right)q_2 - q_2^2$$

$F.O.C$

$$a - q_1 = 4q_2$$

Thus, $q_2^* = \frac{1}{4}\left(a - q_1\right)$

Follower(Firm 1)'s problem

$$\max\left(a - q_1 - \frac{1}{4}\left(a - q_1\right)\right)q_2 - q_1^2$$

$F.O.C$

$$\frac{3}{4}\left(a - q_1\right) - \frac{3}{4}q_1 - 2q_1 = 0$$

$$\therefore q_1^* = \frac{3}{14}a, q_2^* = \frac{11}{56}a$$

$$p^* = a - \frac{3}{14}a - \frac{11}{56}a = \frac{33}{56}a$$

(b)

Firm 2's supply f'n

$$p = MC = 2q_2 \Rightarrow q_2 = \frac{1}{2}p$$

Firm 1's residual demand

$$R\left(p\right) = D\left(p\right) - S\left(p\right) = a - \frac{3}{2}p = q_1$$

Firm 1's problem

$$\max\left(a - \frac{3}{2}p\right)p - \left(a - \frac{3}{2}p\right)^2$$

$F.O.C.$

$$a - 3p + 3a - \frac{9}{2}p = 0$$

$$\therefore p^* = \frac{8}{15}a, q_1^* = \frac{1}{5}a, q_2^* = \frac{4}{15}a$$

9.

(a) Monopolist's problem

$$\max_{q,I}\left(a-bq\right)q-\left(c_0-\beta\sqrt{I}\right)q-I$$

$F.O.C.$

$$\frac{\partial\pi}{\partial q}=a-2bq-\left(c_0-\beta\sqrt{I}\right)=0$$

$$\frac{\partial\pi}{\partial I}=\frac{\beta}{2\sqrt{I}}q-1=0$$

Thus,

$$q^M=\frac{2\left(a-c_0\right)}{4b-\beta^2},I^M=\left(\frac{\beta\left(a-c_0\right)}{4b-\beta^2}\right)^2$$

$$\frac{\partial q^M}{\partial\beta}=\frac{\beta\left(a-c_0\right)}{\left(4b-\beta^2\right)^2}>0$$

$\therefore q^*$ will increase as β is higher

(b) Social planner's problem

$\max CS-\text{cost}$

welfare=$\int_0^q\left(a-bx\right)dx-\left(c_0-\beta\sqrt{I}\right)q-I$

$F.O.C.$

$$\frac{\partial W}{\partial q}=a-bq-\left(c_0-\beta\sqrt{I}\right)=0$$

Thus, $q^S=\dfrac{2\left(a-c_0\right)}{2b-\beta^2},I^S=\left(\dfrac{\beta\left(a-c_0\right)}{2b-\beta^2}\right)^2$

Since $4b-\beta^2>2b-\beta^2,q^M<q^S$ and $I^M<I^S$