# Engineering Economic Analysis Spring 2019 

## Problem set 1

Due: 2019.04.09

1. $\operatorname{Text}$ (Intermediate) 1.7, 2.6, 4.5
2. The reservation prices of 8 people who want to rent an apartment are given below.

| Person | A | B | C | D | E | F | G | H |
| :--- | :--- | :--- | :--- | :--- | :--- | ---: | :--- | :--- |
| Price | 40 | 25 | 30 | 35 | 10 | 18 | 15 | 5 |

Suppose that there are 5 apartments to be rented and that the city rent-control board sets a maximum rent of $\$ 9$. Further suppose that people A, B, C, D, and E manage to get an apartment, while F, G, and H are frozen out.
A. If subletting is legal - or, at least, practiced—who will sublet to whom in equilibrium? (Assume that people who sublet can evade the city rent-control restrictions.)
B. What will be the maximum amount that can be charged for the sublet payment?
C. If you have rent control with unlimited subletting allowed, which of the consumers described above will end up in the 5 apartments?
D. How does this compare to the market outcome?
3. Nancy Lerner is trying to decide how to allocate her time in studying for her economics course. There are two examinations in this course. Her overall score for the course will be the minimum of her scores on the two examinations. She has decided to devote a total of 1,200 minutes to studying for these two exams, and she wants to get as high an overall score as possible. She knows that on the first examination if she doesn't study at all, she will get a score of zero on it. For every 10 minutes that she spends studying for the first examination, she will increase her score by one point. If she doesn't study at all for the second examination she will get a zero on it. For every 20 minutes she spends studying for the second examination, she will increase her score by one point.
A. Draw a "budget line" showing the various combinations of scores on the two exams that she can achieve with a total of 1,200 minutes of studying. On the same graph, draw two or three "indifference curves" for Nancy. On your graph, draw a straight line that goes through the kinks in Nancy's indifference curves. Label the point where this line hits Nancy's budget with the letter A. Draw Nancy’s indifference curve through this point.
B. Write an equation for Nancy's budget line.
C. Write an equation for the line passing through the kinks of Nancy's indifference curves.
D. Solve these two equations in two unknowns to determine the intersection of these lines.
E. Given that she spends a total of 1,200 minutes studying, Nancy will maximize her overall score by spending how many minutes studying for the first examination and how many minutes studying for the second examination?
4. A consumer has a utility function $u\left(x_{1}, x_{2}\right)=\max \left\{x_{1}, x_{2}\right\}$. What is the consumer's demand function for good 1 ? What is the indirect utility function? What is his expenditure function?
5. Consider the indirect function given by

$$
v\left(p_{1}, p_{2}, m\right)=\frac{m}{p_{1}+p_{2}}
$$

A. What are the demand functions?
B. What is the expenditure function?
C. What is the direct utility function?
6. A consumer has a direct utility function of the form

$$
u\left(x_{1}, x_{2}\right)=u\left(x_{1}\right)+x_{2}
$$

Good 1 is a discrete good; the only possible levels of consumptions of good 1 are $x_{1}=0$ and $x_{2}=1$. For convenience, assume that $u(0)=0$ and $p_{2}=1$.
A. What kind of preference does this consumer have?
B. The consumer will definitely choose $x_{1}=0$ if $p_{2}$ is strictly less than what?
C. What is the algebraic form of the indirect utility function associated with this direct utility function?
7. Find the demanded bundle for a consumer whose utility function is $u\left(x_{1}, x_{2}\right)=x_{1}^{\frac{3}{2}} x_{2}$ and her budget constraint is $3 x_{1}+4 x_{2}=100$.
8. The utility function is $u\left(x_{1}, x_{2}\right)=\min \left\{x_{2}+2 x_{1}, x+2 x_{2}\right\}$.
A. Draw the indifference curve for $u\left(x_{1}, x_{2}\right)=20$. Shade the area where $u\left(x_{1}, x_{2}\right) \geq 20$.
B. For what values of $p_{1} / p_{2}$ will the unique optimum be $x_{1}=0$ ?
C. For what values of $p_{1} / p_{2}$ will the unique optimum be $x_{2}=0$ ?
D. If neither $x_{1}$ nor $x_{2}$ is equal to zero, and the optimum is unique, what must be the value of $x_{1} / x_{2}$ ?
9. (2018 Mid-term) Consider a consumer with utility function $u\left(x_{1}, x_{2}\right)=\ln x_{1}+x_{2}$, where $x_{1}$ denotes the gallons of gas and $x_{2}$ represents all other goods. The price of good 2 is therefore normalized to one, $p_{2}=1$. The value tax for the good 1 is imposed with a tax rate of $t \in[0,1]$. Assume that the consumer's income is $m>0$.
A. Find the ordinary demand functions for the both goods, denoting $x_{i}^{*}$, and distinguish the case in whi ch $m>1$ and that when $m \leq 1$.
B. Now the government is considering implementing either of the following policies: (1) reduce the tax on gas, from $t$ to $t-\alpha$; or (2) maintain the tax at $t$ but give a subsidy of $S$ dollars to the consumer equal to the tax revenue collected by the tax on gas. Suppose that the consumer is relatively rich such that income satisfies $m>1$. Under which condition of $\alpha$ does the consumer prefer the first policy

