Engineering Economic Analysis

Spring 2019

Problem set 3

Due: 2019.05.28

1. What is the elasticity of substitution for the general CES technology $y = \left(a_1 x_1^{\rho} + a_2 x_2^{\rho}\right)^{1/\rho}$ when $a_1 \neq a_2$?

2. Consider a perfectly competitive industry.
   (a) Find a short-run supply curve of a firm in this industry whose short-run cost function is estimated as $c(q) = 0.04q^3 - 0.8q^2 + 10q + 5$.
   (b) Suppose that the long-run cost function for each firm in this industry is $c(q) = q^3 - 4q^2 + 8q.$ Describe the perfectly competitive industry's long-run supply function.

3. The production function is $f(x) = 20x - x^2$ and the price of output is normalized to 1. Let w be the price of the x-input. We must have $x \geq 0$.
   a. What is the first-order condition for profit maximization if $x > 0$?
   b. For what values of w will the optimal x be zero?
   c. For what values of w will the optimal x be 10?
   d. What is the factor demand function?
   e. What is the profit function?
   f. What is the derivative of the profit function with respect to w?

4. Consider the technology described by y=0 for $x \leq 1$ and $y = \ln x$ for x>1. Calculate the profit function for this technology.

5. A firm has two plants with cost functions $c_1(y_1) = y_1^2 / 2$ and $c_2(y_2) = y_2$. What is the cost function for the firm?

6. A firm has a cost function

$$c(y) = \begin{cases} y^2 + 1 & \text{if } y > 0 \\ 0 & \text{if } y = 0 \end{cases}$$

   Let p be the price of output, and let the factor prices be fixed. If p=2 how much will the firm will produce? If p=1 how much will the firm will produce? What is the profit function of this firm?

7. For each cost function determine if it is homogeneous of degree one, monotonic, concave and/or continuous. If it is, derive the associated production function.
    a. $c(\mathbf{w}, y) = y^{1/2}(w_1 w_2)^{3/4}$
    b. $c(\mathbf{w}, y) = y\left(w_1 + \sqrt{w_1 w_2} + w_2\right)$
    c. $c(\mathbf{w}, y) = y\left(w_1 e^{-w_1} + w_2\right)$
    d. $c(\mathbf{w}, y) = y\left(w_1 - \sqrt{w_1 w_2} + w_2\right)$
    e. $c(\mathbf{w}, y) = \left(y + \frac{1}{y}\right)\sqrt{w_1 w_2}$

8. (Final exam. of 2018) Find the cost function for the firms with following production function: $f(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$.

9. (Final exam. of 2018) A firm can produce one output (y) using two inputs (x_1, x_2) by means of two different technologies. Technology 1 is represented by production function $y = \min\{x_1, x_2\}$, while technology 2 is represented by the production function $y = (x_1 + x_2)/3$. Output price is $p > 0$ and input prices are $w_1, w_2 \geq 0$.

(a) Characterize the returns to scale of each technology (Increasing, Constant, Decreasing).

(b) Suppose that the firm wants to produce a specific amount of output $\bar{y}$. For which values of w_1 will the firm use technology 1?

10. (Final exam. of 2018) Consider a perfectly competitive market with heterogeneous goods. In particular, suppose that every firm $i \in N$ faces an inverse demand function

$$p_i(q_i, q_{-i}) = \frac{\theta q_i^{\beta-1}}{\sum_{j=1}^{N} q_j^{\beta}}$$

where q_i denotes firm i's output, q_{-i} the output of all other firms, i.e., $q_{-i} = (q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_N)$, θ is a positive constant, and parameter $\beta \in (0,1]$ captures the degree of substitutability among heterogeneous goods such that as goods become more differentiated, the value of β will be higher. In addition, assume that every firm faces the same cost function $c(q_i) = F + cq_i$.

(a) Find the individual firm i's production level in the symmetric equilibrium, i.e., $q_i^* = q^*$ for all i .

(c) Using the result of (a), determine '*True*' or '*False*' of the following statement with a proper reasoning. "When others are being equal, the equilibrium output level in hand-made food market is larger than that of fast food market."