# Engineering Economic Analysis <br> Spring 2019 

## Problem set 4

Due: 2019.06.18(Tue)

1. An industry consists of a large number of firms, each of which has a cost function of the firm

$$
c\left(w_{1}, w_{2}, y\right)=\left(y^{2}+1\right) w_{1}+\left(y^{2}+2\right) w_{2}
$$

(a) Find the average cost curve of a form and describe how it shifts as the factor price $w_{1} / w_{2}$ changes.
(b) Find the short-run supply curve of an individual firm.
(c) Find the long-run industry supply curve.
2. Consider an industry with the following structure. There are 50 firms that behave in a competitive manner and have identical cost functions given by $c(y)=y^{2} / 2$. There is one monopolist that has 0 marginal costs. The demand curve for the product is given by

$$
D(p)=1000-50 p
$$

(a) What is the monopolist's profit-maximizing output?
(b) What is the monopolist's profit-maximizing price?
(c) How much does the competitive sector supply at this price?
3. U.S. consumers have a demand function for umbrellas which has the form $D(p)=90-p$. Umbrellas are supplied by U.S. firms and U.K. firms. For simplicity, assume that there is a single representative firm in each country that behaves competitively. The cost function for producing umbrellas is given by $c(y)=y^{2} / 2$ in each country.
(a) What is the aggregate supply function for umbrellas?
(b) What is the equilibrium price and quantity sold?
(c) Now the domestic industry lobbies for protection and Congress agrees to put a $\$ 3$ tariff on foreign umbrellas. What is the new U.S. price for umbrellas paid by the consumers?
(d) How many umbrellas are supplied by foreign firms and how many are supplied by domestic firms?
4. Suppose marginal costs are constant at $\mathrm{c}>0$ and that the demand function is given by

$$
D(p)=\left\{\begin{array}{cc}
10 / p \text { if } p \leq 20 \\
0 & \text { if } p>20
\end{array}\right.
$$

What is the profit-maximizing price?
5. There is a single monopolist whose technology exhibits constant marginal costs, i.e., $c(y)=c y$. The market demand curve exhibits constant elasticity, $\varepsilon$. There is an ad valorem tax on the price of the good cold so that when the consumer pays a price $P_{D}$, the monopolist receives a price of $P_{S}=(1-\tau) P_{D}$. (Here $P_{D}$ is the demand price facing the consumer and $P_{S}$ is the supply price facing the producer.)
The taxing authority is considering changing the ad valorem tax to a tax on output, $t$, so that we will have $P_{D}=P_{S}+t$. Calculate the output tax $t$ that is equivalent to the ad valorem $\operatorname{tax} \tau$ in the sense that the final price faxing the consumer is the same under either scheme.
6. A monopolist sells in two markets. The demand curve for the monopolist's product is $x_{1}=a_{1}-b_{1} p_{1}$ in market 1 and $x_{2}=a_{2}-b_{2} p_{2}$ in market 2, where $x_{1}$ and $x_{2}$ are the quantities sold in each market, and $p_{1}$ and $p_{2}$ are the prices charged in each market. The monopolist haw zero marginal costs. Note that although the monopolist can charge different prices in the two markets, it must sell all units within a market at the same price.
(a) Under what conditions on the parameters $\left(a_{1}, b_{1}, a_{2}, b_{2}\right)$ will the monopolist optimally choose not to price discriminate? (Assume interior solutions.)
(b) Now suppose that the demand functions take the form $x_{i}=A_{i} p_{i}^{-b_{i}}$, for $i=1,2$, and the monopolist has some constant marginal cost of $c>0$. Under what conditions will the monopolist choose not to price discriminate? (Assume interior solutions.)
7. Consider an industry with the following structure. There are 50 firms that behave in a competitive manner and have identical cost functions given by $c(y)=y^{2} / 2$. There is one monopolist that has 0 marginal costs. The demand curve for the product is given by

$$
D(p)=1000-50 p
$$

(a) What is the supply curve of one of the competitive firms?
(b) What is the total supply from the competitive sector?
(c) If the monopolist sets a price $p$, how much output will it sell?
(d) What is the monopolist's profit-maximizing output?
(e) What is the monopolist's profit-maximizing price?
(f) How much will the competitive sector provide at this price?
(g) What will be the total amount of output sold in this industry?
8. (Final exam. of 2018) Suppose that the inverse demand function is given by $P(Q)=a-Q$, and the cost function of each firm $i$ is $C_{i}\left(q_{i}\right)=q_{i}^{2}$.
(a) Find the equilibrium price and quantities of quantity-leadership duopoly where firm 1 is leader and firm 2 is follower.
(b) Find the equilibrium price and quantities of price-leadership duopoly game where firm 1 is leader and firm 2 is follower.
9. (Final exam. of 2018) Consider a monopolist with inverse demand function $p(q)=a-b q$. The monopolist makes two choices: how much to invest in cost reduction, $I$, and how many to produce, $q$. If the monopolist invests $I$ in cost reduction, his constant per-unit cost of production is $c(I)=c_{0}-\beta \sqrt{I}$, where $c_{0}>0$ is the initial marginal cost, and $\beta>0$ denotes the effectiveness of cost-reducing investment. For simplicity, assume that $a>c$, and $b>\beta^{2} / 2$.
(a) If the monopolist is a selfish profit-maximizer, determine whether the optimal level of output will increase or decrease as the effectiveness of cost-reducing investment is higher.
(b) Suppose that the monopolist is a benevolent social planner who wants to maximize total surplus which is equal to the consumer surplus minus production \& investment costs. Then show that this benevolent monopolist invests and produces more than the selfish profit-maximizer would.

