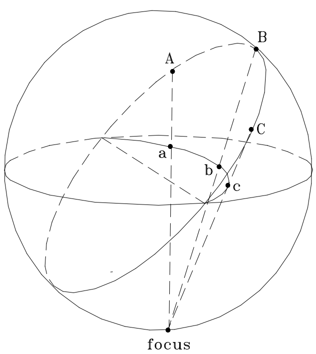
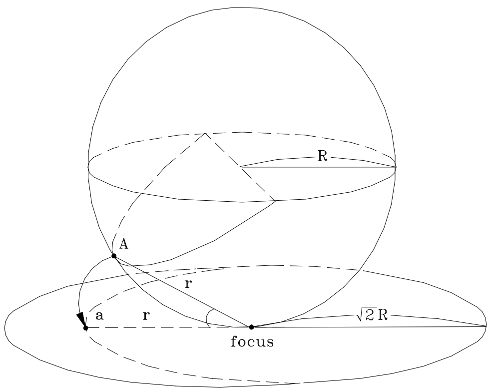
Stereographic projection

Orientation of a discontinuity in 3D space can be depicted with an arc or point in 2D plane by projection methods. Let the discontinuity pass through a center of sphere having an empty space inside and draw lines from a focal point at the top or bottom of the sphere to the intersection points of the discontinuity and the sphere. Then the intersection of the lines and a horizontal plane passing through the sphere center is a circle in the horizontal plane. This circle is a result of stereographic or equal-angle (Wulf) projection of the circle on the sphere (Fig. 1(a)). If the points of circle on the sphere are rotated round the focus until they intersect a horizontal plane, the curved trace on the horizontal plane is a result of equal-area or Lambert (Schmidt) projection (Fig.1(b)).

The equal-angle projection is relatively simple in its mathematical expression and frequently applied to engineering fields such as block theory. The equal-area projection is preferred for the statistical analysis of joint poles. Projection of a horizontal circle by the equal-area projection has a greater radius ($\sqrt{2}$ times) than the original one which means that the projected circle should be resized to a contracted circle at final step.



(a) Equal-angle projection of the upper reference hemisphere



(b) Equal-area projection of the lower reference hemisphere

Fig. 1 Projection of a great circle on the reference hemisphere.

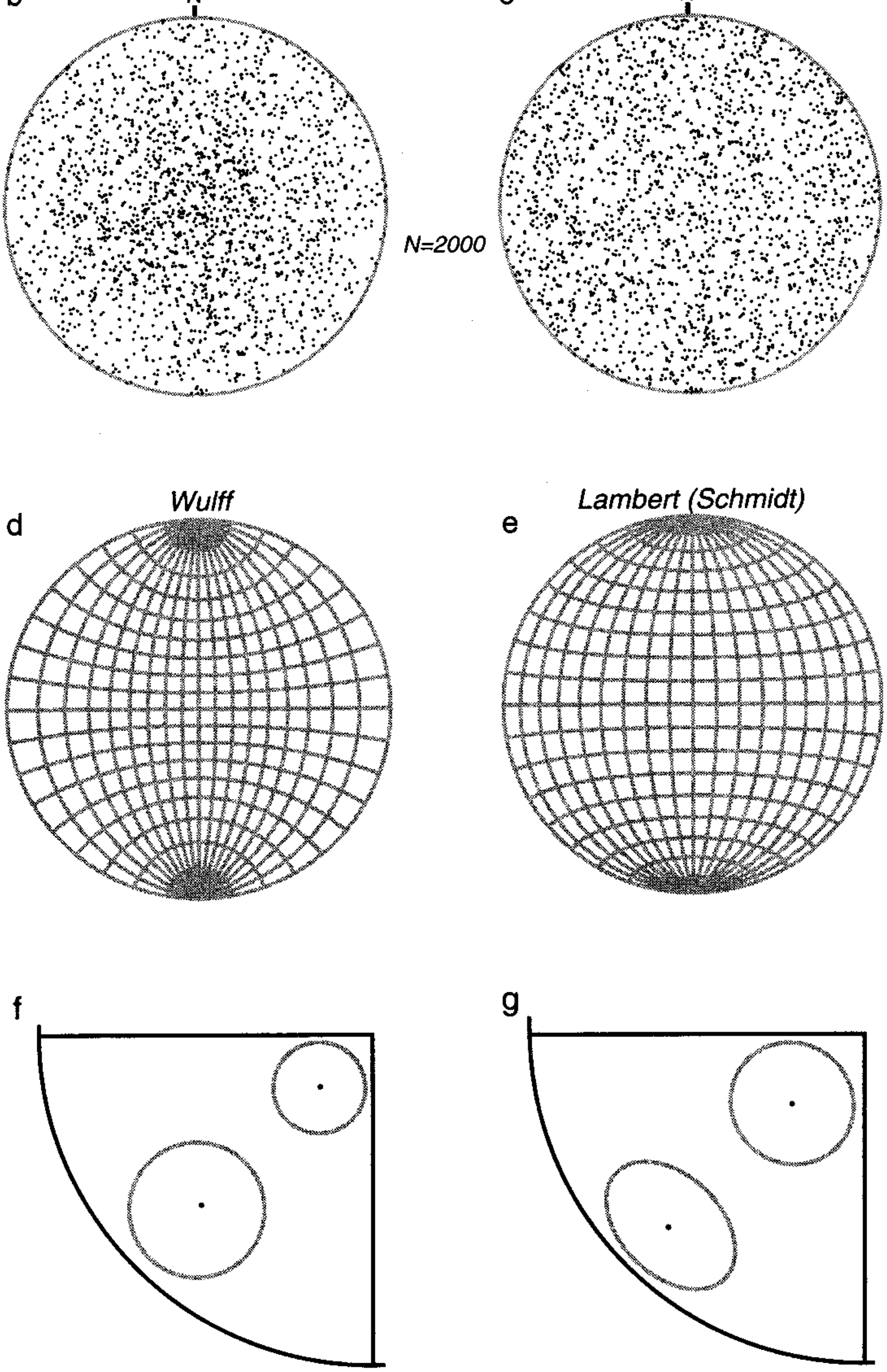


Fig. 2 Bias due to projection: Equal-area projection (L) and equal-area projection (R) (R.J.Lisle et al., 2004)

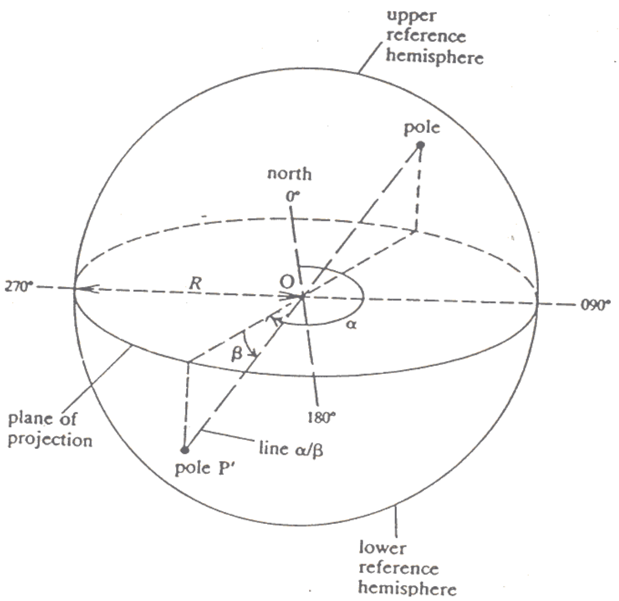


Fig. 3 The reference sphere, intersected by a line of trendα and plungeβ (after Priest, 1985).

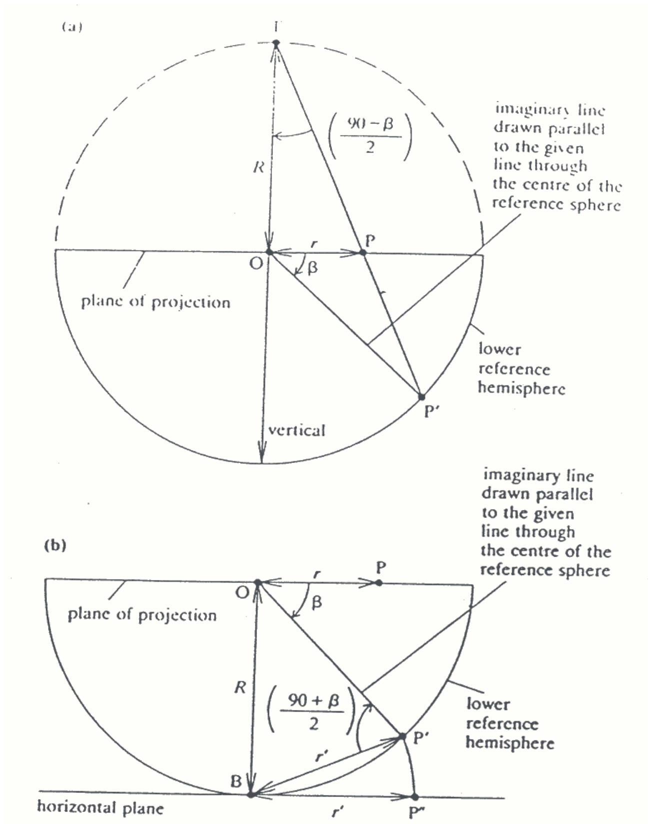


Fig. 4 Vertical sections through the center of the lower reference hemisphere (a) eqaul-angle projection and (b) equal-area projection (after Priest, 1985).

$$r = R\tan\left(\frac{90^\circ - \beta}{2}\right)$$

$$x = R\cos\alpha\,\tan\left(\frac{90^\circ - \beta}{2}\right)$$

$$y = R\sin\alpha\,\tan\left(\frac{90^\circ - \beta}{2}\right)$$

$$r' = 2R\cos\left(\frac{90^\circ + \beta}{2}\right)$$

$$r = \sqrt{2}\,R\cos\left(\frac{90^\circ + \beta}{2}\right)$$

$$x = \sqrt{2}\,R\cos\alpha\,\cos\left(\frac{90^\circ + \beta}{2}\right)$$

$$y = \sqrt{2}\,R\sin\alpha\,\cos\left(\frac{90^\circ + \beta}{2}\right)$$

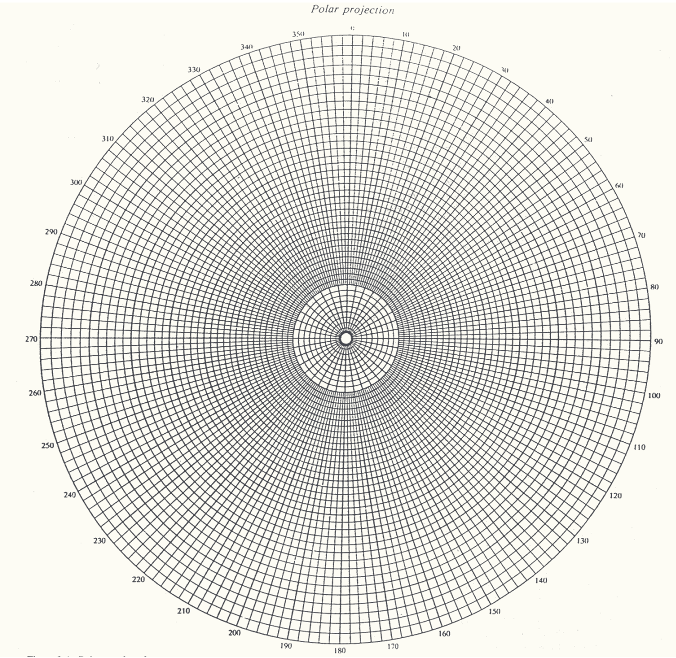


Fig. 5 Polar equal-angle net(after Priest, 1985).

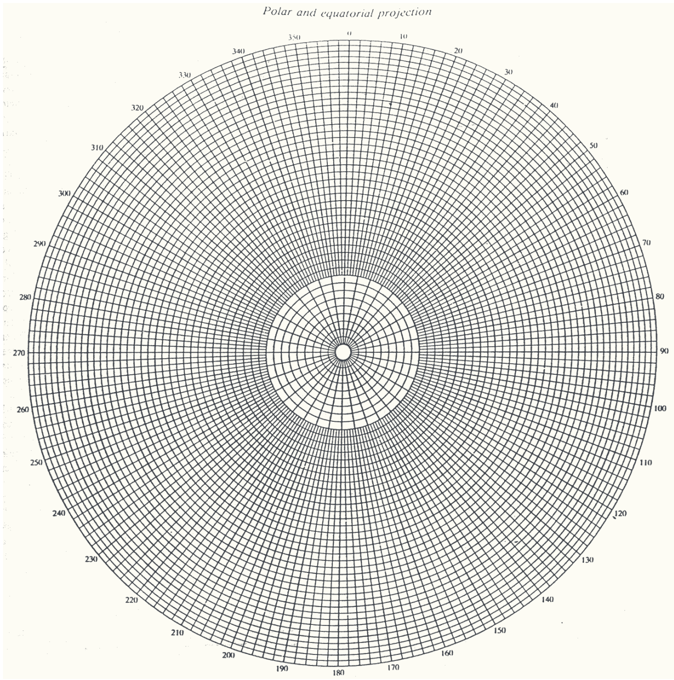


Fig. 6 Polar equal-area net(after Priest, 1985).

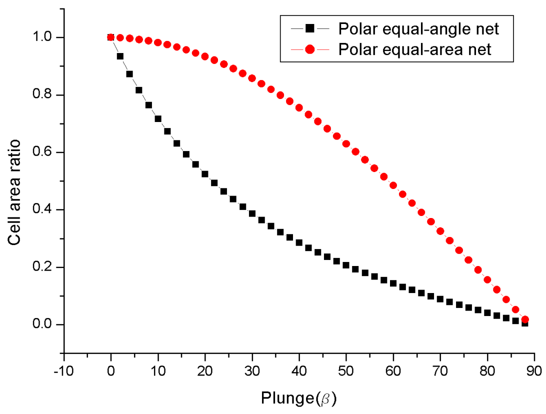


Fig. 7 Cell area ratios of the polar equal-angle(area) net.

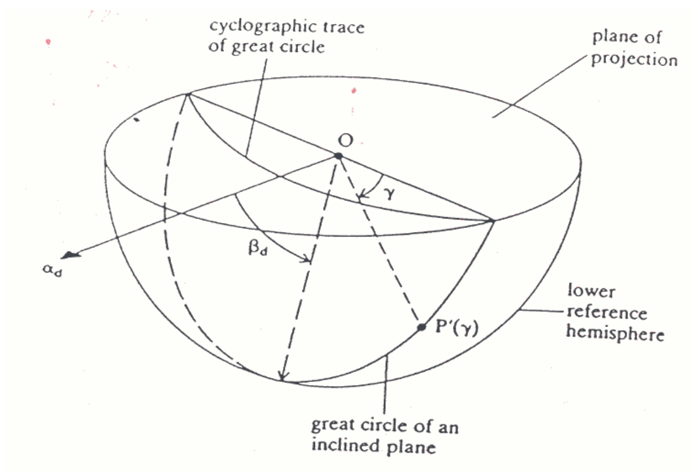


Fig. 8 The cyclographic trace of a great circle
(after Priest, 1985).

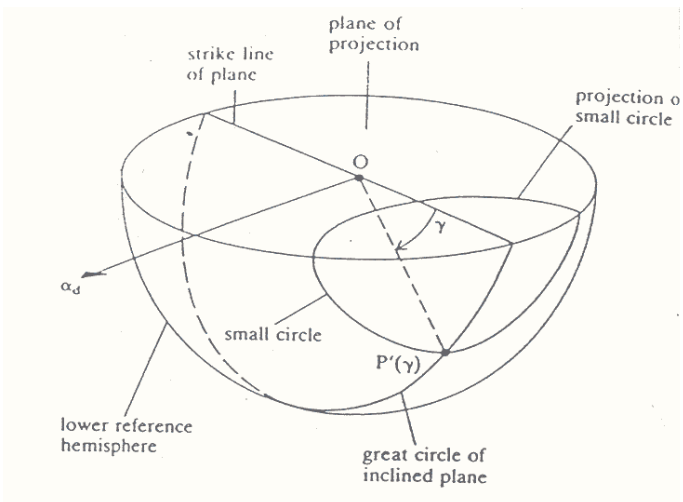


Fig. 9 The definition of a small circle(after Priest, 1985).

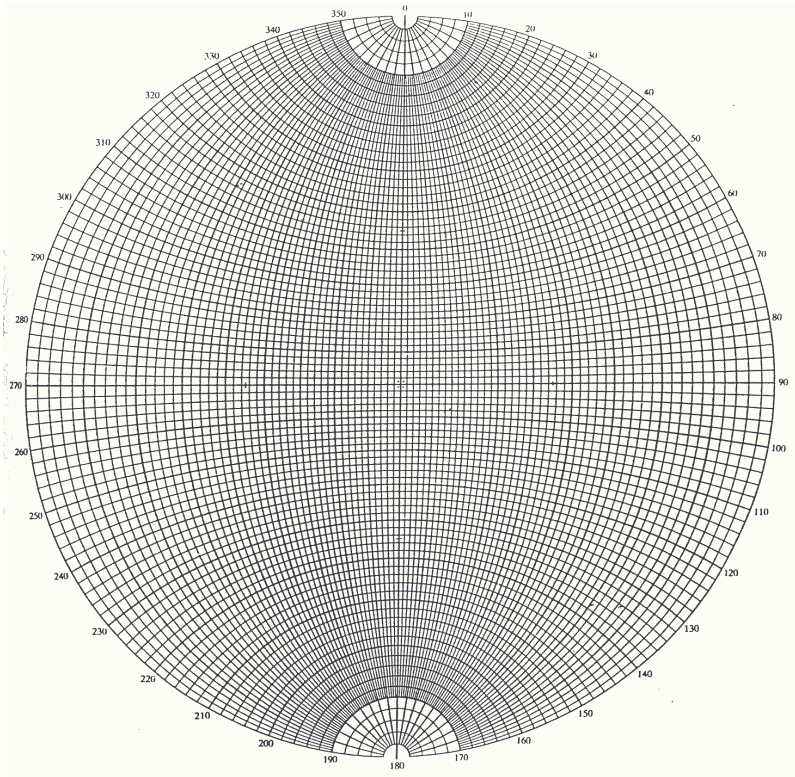


Fig. 10 Equatorial equal-angle net(after Priest, 1985).

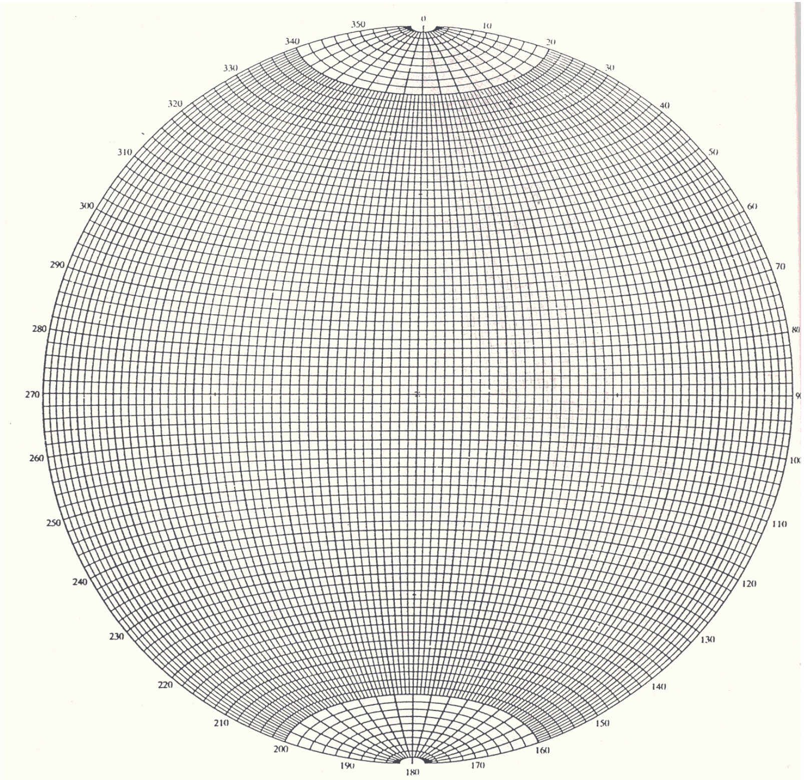


Fig. 11 Equatorial equal-area net(after Priest, 1985).

Problem 1) Draw poles of 286 discontinuities whose dip direction and dip are given in the table by applying the equal-area projection to the upper and lower hemispheres (two figures).

13	63	77	58	110	82	235	53	20	25	15	90	13	82	337	70
193	65	281	60	248	82	210	47	157	34	24	87	150	83	341	70
340	65	276	54	291	80	286	42	62	26	136	87	155	80	344	70
345	87	255	50	245	78	254	25	97	28	141	86	186	83	344	70
346	86	247	32	279	75	45	87	129	26	143	87	186	83	346	70
350	86	37	85	225	67	67	87	167	30	147	85	327	80	353	70
350	90	53	87	217	65	220	86	121	20	147	85	336	82	342	88
353	85	93	86	263	63	240	87	25	21	150	87	338	80	360	70
355	90	237	87	232	60	255	87	33	20	162	88	342	80	355	60
347	62	360	70	230	55	274	87	57	24	163	85	350	83	304	53
350	65	10	64	301	52	45	83	70	20	167	86	352	80	8	47
9	56	167	57	218	45	84	80	115	25	175	87	355	80	16	34
28	58	313	58	249	30	232	82	260	20	182	86	355	84	308	27
149	60	328	60	44	90	257	80	126	20	314	90	192	77		
230	86	250	87	57	87	84	75	127	23	280	90	343	60		
243	88	270	90	116	86	247	75	133	20	62	83	328	78		
257	86	287	90	240	85	302	78	145	25	85	80	346	79		
250	77	73	83	286	90	248	70	158	22	232	82	351	75		
265	73	105	80	65	80	229	64	214	18	264	80	354	75		
255	68	244	81	101	80	268	65	267	22	218	76	354	75		
244	65	276	82	234	82	244	58	325	20	314	88	355	75		
297	65	245	79	252	86	240	55	77	18	315	90	357	78		
270	57	255	77	270	88	217	47	111	19	325	86	360	77		
270	54	270	74	30	80	291	45	153	16	326	90	334	72		
230	50	90	65	83	82	202	75	180	0	327	90	344	74		
216	33	255	65	327	83	168	46	28	90	328	90	356	74		
199	43	228	60	328	80	177	46	50	90	341	90	357	74		
36	88	292	57	330	84	83	42	74	90	342	85	6	70		
50	90	297	55	331	80	60	37	7	15	106	44	8	70		
86	86	216	43	322	82	118	37	81	13	176	43	10	68		
230	85	275	35	210	80	154	40	85	12	205	36	15	67		
250	87	41	87	255	82	177	36	132	6	343	85	140	67		
258	87	56	87	297	80	181	40	190	7	345	85	147	70		
26	84	109	85	247	78	133	35	7	90	345	87	161	70		
240	77	240	85	285	75	30	34	10	90	345	88	304	70		
251	78	251	86	242	70	81	32	12	86	356	87	314	70		
270	73	270	85	220	65	105	32	13	90	356	85	333	67		
277	66	296	90	266	65	119	32	13	85	358	87	334	66		
247	63	75	80	233	60	126	31	14	87	360	88	337	70		

Problem 2) Delimit sets from the joints in the table by using the algorithm of R.J Shanley & M.A. Mahtab.