Term Project: Problem Set B

Instructor: Prof. Chongam Kim
TA: Hyunji Kim
Due: May 24th
Room: 301-1256

The discretization of the **1-D Euler equation** is considered to compute the following five inviscid compressible flows.

Case1) ***Modified*** **Sod's shock-tube problem:**
$(\rho, u, p)_L = (1.0, 0.75, 1.0)$ if $x \le 0.3$; $(\rho, u, p)_R = (0.125, 0.0, 0.1)$ if $x > 0.3$.
100 cells, CFL=0.9 at t=0.2

Case2) Supersonic expansion (*123* problem):
$(\rho, u, \rho E)_L = (1.0, -2.0, 3.0)$ if $x \le 0.5$; $(\rho, u, \rho E)_R = (1.0, 2.0, 3.0)$ if $x > 0.5$.
100 cells, CFL=0.5 at t=0.15

Case3) Blast wave problem:
$(\rho, u, p)_L = (1.0, 0.0, 1000.0)$ if $x \le 0.5$; $(\rho, u, p)_R = (1.0, 0.0, 0.01)$ if $x > 0.5$.
100 cells, CFL=0.6 at t=0.012

Case4) Test for *two strong shock waves traveling towards each other*:
$(\rho, u, p)_L = (5.99924, 19.5975, 460.894)$ if $x \le 0.4$;
$(\rho, u, p)_R = (5.99242, -6.19633, 46.0950)$ if $x > 0.4$.
100 cells, CFL=0.8 at t=0.035

Case5) Test for *slowly-moving contact discontinuities*:
$(\rho, u, p)_L = (1.0, -19.59745, 1000.0)$ if $x \le 0.8$;
$(\rho, u, p)_R = (1.0, -19.59745, 0.01)$ if $x > 0.8$.
100 cells, CFL=0.6 at t=0.012

The numerical methods to be tested are:
1) First-order flux functions
- FVS: van Leer's FVS [2]
- FDS: Roe's FDS [3], RoeM2 [4]
- Hybrid: AUSM+ [5], AUSMPW+ [6]

2) High-order interpolation version of 1)
- TVD methods using flux limiters [7]
- MUSCL approach [8]
- WENO [9]

* Runge-Kutta time integration is preferred for methods 2).
The 3rd-order TVD R-K method for $q_t = L(q)$ is given by

$$q^{(1)} = q^n + \Delta t L\left(q^n\right),$$
$$q^{(2)} = \frac{3}{4}q^n + \frac{1}{4}q^{(1)} + \frac{1}{4}\Delta t L\left(q^{(1)}\right),$$
$$q^{n+1} = \frac{1}{3}q^n + \frac{2}{3}q^{(2)} + \frac{2}{3}\Delta t L\left(q^{(2)}\right).$$

For each problem,

i) Plot density, pressure, velocity and total energy at the given time.
ii) Discuss accuracy, efficiency and robustness of each method.
iii) CFL number and mesh points may be changed to see their effects.
iv) Each computed result can be compared with fine grid computations and/or analytical solutions.

If you have enquiries or need help with your term project, please send an email to TA.

References

[1] E. F. Toro, “Riemann solvers and numerical methods for fluid dynamics, 3rd edition,” Springer, 2009. **(book)**
[2] B. van Leer, “Flux-vector splitting for the Euler equations”, in 8th International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, vol. 170, 1982.
[3] P. Roe, “Approximate Riemann solvers, parameter vectors, and difference schemes,” Journal of Computational Physics, vol. 43, no. 2, pp. 357 – 372, 1981.
[4] S.-s. Kim, C. Kim, O.-H. Rho, and S. K. Hong, “Cures for the shock instability: Development of a shock-stable Roe scheme,” Journal of Computational Physics, vol. 185, no. 2, pp. 342 – 374, 2003.
[5] M.-S. Liou, “A sequel to AUSM: AUSM+,” Journal of Computational Physics, vol. 129, no. 2, pp. 364 – 382, 1996.
[6] K. H. Kim, C. Kim, and O.-H. Rho, “Methods for the accurate computations of hypersonic flows: I. AUSMPW+ scheme,” Journal of Computational Physics, vol. 174, no. 1, pp. 38 – 80, 2001.
[7] P. K. Sweby, “High resolution schemes using flux limiters for hyperbolic conservation laws,” SIAM journal on numerical analysis, vol. 21, no. 5, pp. 995–1011, 1984.
[8] B. van Leer, “Towards the ultimate conservative difference scheme. V. a second-order sequel to Godunov’s method,” Journal of Computational Physics, vol. 32, no. 1, pp. 101 – 136, 1979.
[9] X.-D. Liu, S. Osher, and T. Chan, “Weighted essentially non-oscillatory schemes,” Journal of Computational Physics, vol. 115, no. 1, pp. 200 – 212, 1994.