Term Project: Problem Set D

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Due: June 10th
Room: 301-1256

- Governing equation: One-dimensional scalar conservation law
- Basic discretization: Discontinuous Galerkin method (modal basis)
- Flux function: Upwind method
- Quadrature rule: Gauss-Lobatto
- Time integration method: Euler explicit method, 5-stage 4th-order accurate SSP Runge-Kutta method
- Order of solution approximation (***N***): *P1, P2, P3, P4*
- Mesh size (***K***): $\Delta x = 0.1, 0.05, 0.025, 0.0125, 0.00625$ ($K = 2/\Delta x$)
- Target time: $t = 12.0, 24.0, 48.0, 192.0, 384.0$
  (Use CFL condition to determine the time-step size.)

< Linear advection equation >

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0, \quad a = 0.5, \quad -1 \le x \le 1.$$

Initial conditions: 1) Gaussian wave $u(x,0) = e^{-300x^2}$

2) Sine wave $u(x,0) = 1 + 0.5\sin(\pi x)$

Boundary condition: Periodic condition (i.e., $u(-1,t) = u(1,t)$)

1. Conduct the error analysis for each combination of (N, K) and obtain convergence rate for each N. Set target time as 12.0. Compare the results of two time integration methods.
2. Compare the scaled computational costs of all combinations of (N, K) at target time 12.0. Let the computational cost of (N, K)=(1, 20) be unity.
3. Select at least three combinations of (N, K) for similar $(N+1)\times K$ value, and check the magnitudes of errors at five different target times given above. Use 5-stage 4-th order accurate SSP R-K method.

Refer to supplement for an example.