

< 전자장2 Midterm-exam#2 Solution >

$$\begin{aligned}
 1. \text{ a) } \sin \theta_{B\parallel} &= \frac{1}{\sqrt{1 + (\frac{\epsilon_1}{\epsilon_2})}} \quad (\text{when } \mu_1 = \mu_2) \\
 &= \frac{1}{\sqrt{1 + (\frac{\epsilon_0}{3\epsilon_0})}} = \frac{\sqrt{3}}{2} \\
 \therefore \theta_{B\parallel} &= 60^\circ
 \end{aligned}$$

$(\theta_{B\perp}$ does not exist for non-magnetic medium)

$$\begin{aligned}
 \frac{\sin \theta_t}{\sin \theta_i} &= \frac{\sin \theta_t}{\sin \theta_{B\parallel}} = \sqrt{\frac{\epsilon_0}{3\epsilon_0}} = \frac{1}{\sqrt{3}} \\
 \sin \theta_t &= \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{2} = \frac{1}{2} \\
 \therefore \theta_t &= 30^\circ
 \end{aligned}$$

b) For perpendicular polarization,

$$\begin{aligned}
 \bar{E}_t(x, z) &= \hat{y} E_{t0} e^{-j\beta_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)} \\
 \bar{H}_t(x, z) &= \frac{E_{t0}}{\eta_2} (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t) e^{-j\beta_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)}
 \end{aligned}$$

(Parallel polarization wave does not affect E_y, H_x component.)

$$\begin{aligned}
 \Rightarrow -\frac{E_y}{H_x} &= -\frac{E_{t0} e^{-j\beta_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)}}{-\frac{E_{t0}}{\eta_2} \cos\theta_t e^{-j\beta_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)}} = \frac{\eta_2}{\cos\theta_t} \\
 &= \frac{\eta_0/\sqrt{3}}{\cos 30^\circ} = \eta_0 \frac{2}{3} = 120\pi \frac{2}{3} = 80\pi \\
 (\because \eta_2 &= \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{1}{\sqrt{3}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{3}})
 \end{aligned}$$

c) Brewster angle로 푸기

$$\left(\frac{P_{out}}{P_{in}}\right)_\parallel = 1 \quad : \text{Total transmission}$$

$$\left(\frac{P_{out}}{P_{in}}\right)_\perp = \left(\frac{3}{4}\right)^2 = 0.563$$

But,

$$\left(\frac{P_{out}}{P_{in}}\right)_\perp = (1 - \Gamma_{air-slab}^2)^2 = 0.563$$

$$\Gamma_{air-slab} = \frac{Z_g - Z_a}{Z_g + Z_a} = \frac{80\pi - 240\pi}{80\pi + 240\pi} = -\frac{1}{2}$$

2. a) $\theta_t + \theta_c = \frac{\pi}{2}$ where, $\begin{cases} \theta_t : \text{angle of transmission(refraction)} \\ \theta_c : \text{critical angle} \end{cases}$

From Snell's law,

$$\begin{aligned} \frac{\sin \theta_t}{\sin \theta_a} &= \frac{n_0}{n_1} \\ \Rightarrow n_0 \sin \theta_a &= n_1 \sin \theta_t \\ &= n_1 \sin\left(\frac{\pi}{2} - \theta_c\right) \\ &= n_1 \cos \theta_c \\ &= n_1 \sqrt{1 - \sin^2 \theta_c} \quad (\sin \theta_c = \frac{n_2}{n_1}) \\ \Rightarrow \sin \theta_a &= \frac{1}{n_0} \sqrt{n_1^2 - n_2^2} \quad (\text{free space: } n_0 = 1) \\ &= \sqrt{n_1^2 - n_2^2} \\ \therefore \theta_a &= \sin^{-1}(\sqrt{n_1^2 - n_2^2}) \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \theta_a = \sin^{-1}(\sqrt{n_1^2 - n_2^2}) \\
 &= \sin^{-1}(\sqrt{2^2 - (1.74)^2}) \\
 &\approx 80.4^\circ
 \end{aligned}$$

$$\text{N.A} = \sin \theta_a \approx 0.986$$

3. open circuit $Z_L \rightarrow \infty : Z_{\text{io}} = Z_0 \coth \gamma l$
short circuit $Z_L \rightarrow 0 : Z_{\text{is}} = Z_0 \tanh \gamma l$

$$\begin{cases} \gamma = \alpha + j\beta \\ \gamma l = (\alpha + j\beta) l = \alpha l + j\beta l \end{cases}$$

$$\begin{aligned}
 \text{a)} \quad & Z_{\text{is}} = Z_0 \tanh \gamma l \approx Z_0 (\gamma l) \\
 & \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \\
 & Z_0 = \sqrt{\frac{(R + j\omega L)}{G + j\omega C}} \\
 & \therefore Z_{\text{is}} = (R + j\omega L) l
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & Z_{\text{io}} = Z_0 \coth \gamma l \approx \frac{Z_0}{\gamma l} \\
 & = \frac{1}{G + j\omega C} = \frac{G - j\omega C}{(G^2 + \omega^2 C^2) l}
 \end{aligned}$$

4. a) The velocity of propagation

$$10^9 t - 5z = \text{const} \Rightarrow u_p = \frac{dz}{dt} = \frac{10^9}{5} = 2 \cdot 10^8 \text{ (m/s)}$$

for lossless transmission line,

$$\begin{aligned}
 u_p &= \frac{1}{\sqrt{LC}}, \quad Z_0 = R_0 = \sqrt{\frac{L}{C}} \\
 \Rightarrow u_p \cdot Z_0 &= \frac{1}{C} = 2 \cdot 10^8 \cdot 300 = 600 \cdot 10^8 \\
 \therefore C &= \frac{1}{600 \cdot 10^8} = \frac{50}{3} \cdot 10^{-12} = \frac{50}{3} (\text{pF/m})
 \end{aligned}$$

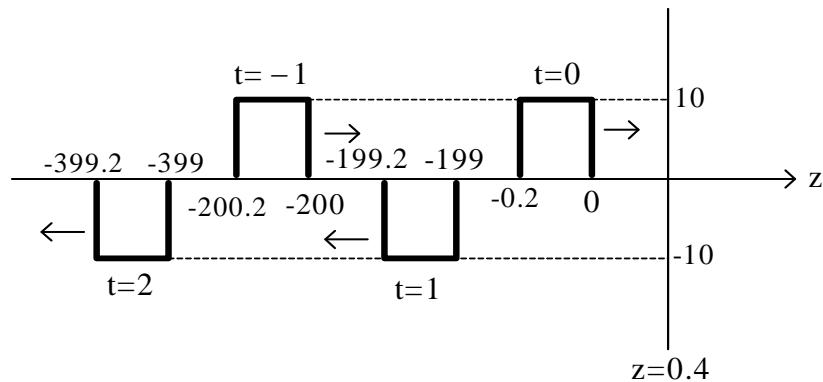
- b) The propagation velocity is $2 \cdot 10^8 \text{ m/s}$,
so wave moves 200m per $1\mu\text{s}$.

$$\text{Meanwhile } \Gamma = \frac{R_L - R_0}{R_L + R_0} = -1,$$

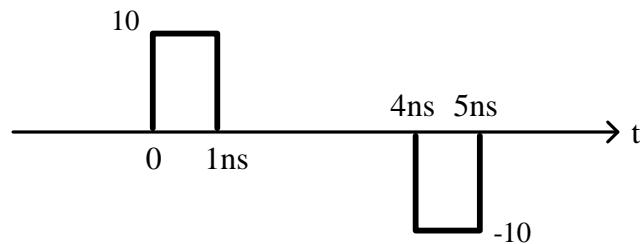
so after reflection the propagating direction and the sign of amplitude is reversed.

When $t = 0 \Rightarrow V^+(z,0) = 10w(-5z)$,
so the width of the square wave is 0.2 ($z = -0.2$ to $z = 0$).

It moves 200m per $1\mu\text{s}$,
and after reflection its direction and sign of amplitude is reversed.



- c) At $z=0 \Rightarrow V^+(0,t) = 10w(10^9 t)$,
so the width of the square is 1ns ($t=0$ to $t = 1\text{ ns}$).
After reflection at the end of the line ($z=0.4$),
reflecting wave comes back after 4ns ($\frac{0.4 \times 2}{2 \cdot 10^8}$).



5. normalized impedance

$$z_L = \frac{Z_L}{Z_0} = \frac{30 + j10}{50} = 0.6 + j0.2$$

a) SWR

- 1) Locate z_L on Smith chart (Point P_1)
- 2) With center at '0', draw a $|\Gamma|$ -circle through P_1 , intersecting OP_{OC} at 1.77
 $\therefore S=1.77$

b) Voltage reflection coefficient

$$\begin{aligned}\Gamma &= |\Gamma| e^{j\theta_r} = \frac{S-1}{S+1} e^{j\theta_r} \\ &= \frac{1.77-1}{1.77+1} e^{j146^\circ} \cong 0.28 e^{j146^\circ}\end{aligned}$$

c) Input impedance

- 1) Draw line OP_1 , intersecting the periphery at P_1' .
 Read 0.046 on 'wavelengths toward generator' scale.
- 2) Move clockwise by 0.101λ to 0.147 (Point P_2').
- 3) Join O and P_2' , intersecting the $|\Gamma|$ -circle at P_2 .
- 4) Read $z_i = 1 + j0.59$ at P_2 .
 $Z_i = 50z_i = 50 + j29.5 \Omega$

d) Input admittance

Extend line P_2P_2O to P_3 . Read $y_i = 0.75 - j43$.

$$Y_i = \frac{1}{50} y_i = 0.015 - j0.009 \text{ (S)}$$

e) There is no voltage minimum on the line, but $V_L < V_i$

