Selected Solutions of Mid-term Exam 2

1. 3) X (is proportional → is <u>not</u> proportional
   or dispersive → nondispersive)

   5) X (inductive → capacitive
   or open-circuited → short-circuited)

   7) X ($\lambda/2 \to \lambda/4$
   or neighboring ··· voltage minima → successive voltage maxima (minima))

   9) X ((1, 0) → (−1, 0)
   or open-circuit, P_{oc} → short-circuit, P_{sc}
   or admittance → impedance)

   14) X (wavelengths → half-wavelengths)

   Others) O

2. a) FIGURE 8-2

   b) (8-1)→(8-3) for $v(z,t)$, (8-4)→(8-5) for $i(z,t)$

   c) (8-8), (8-9) → (8-10)
   where $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$ (8-12)

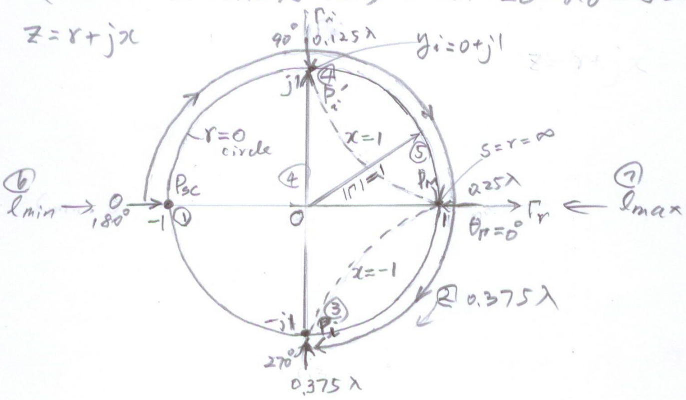
   d) (8-33): $V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$ (V)

   e) (8-35) & (8-38): $Z_0 \equiv \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} \underset{(8\text{-}34)}{=} \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

   f) (8-46) ~ (8-51):
   $\frac{R}{L} = \frac{G}{C}$: distortionless condition in c) and e)
   $\gamma = \sqrt{C/L}\,(R+j\omega L) \Rightarrow \alpha = R\sqrt{C/L}$, $\beta = \omega\sqrt{LC}$
   $Z_0 = R_0 + jX_0 = \sqrt{L/C}$

   g) For matched lines, $\Gamma = 0$ and $V_0^- = 0$ (no reflection wave)
   $\therefore \Gamma \equiv \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \Rightarrow Z_L = Z_0$

3. Given (lossless short-circuited line with $l = 0.1875$ (m) and $Z_0 = R_0 = 5(\Omega)$)



a) ① Locate P_{sc}, and at P_{sc} $\Gamma = \Gamma_r + j\Gamma_i = -1 + j0$
$= |\Gamma| e^{j\theta_\Gamma} = 1\, e^{j180°}$

b) $f = 600\,(\mathrm{MHz}) = 6\times10^8$ Hz $\rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^8} = 0.5$ (m)

$\therefore l = 0.1875\,(m) = \frac{0.1875}{0.5} = 0.375\lambda$

② move P_{sc} along WTG by $\frac{l}{\lambda} = 0.375$ up to P_i at input end.

③ Read z_i at P_i, $z_i = r_i + jx_i = 0 - j1$

$\therefore Z_i = R_0 z_i = 5(0 - j1) = \underline{-j5\,(\Omega)}$

c) ④ move to P_i' diametrically opposite to P_i to find y_i

Read $y_i = 0 + j1 \Rightarrow Y_i = \frac{y_i}{Z_0} = \frac{j1}{5} = \underline{j\,0.2\ (S)}$

d) ⑤ Draw a circle of radius $\overline{OP_{sc}}$: $|\Gamma| = 1.0$
and read r at P_M where the $|\Gamma| = 1.0$ circle intersects with the positive-real axis.
$\Rightarrow \underline{s = r = \infty}$

e) ⑥ At P_{sc} where the voltage is a minimum distance from the s.c. end $\Rightarrow \underline{l_{min} = 0}$

⑦ At P_M where the voltage is a maximum distance from the s.c. end $\Rightarrow l_{max} = 0.25\lambda$
$= 0.25 \times 0.5$
$= \underline{0.125\ (m)}$

4. Given (air-filled $a \times b$ rectang. waveguide
with ϵ_0, μ_0, $a=\sqrt{3}b$, $f=3.6\,GHz$, for TM wave
($H_z = 0$, $E_z = E_z^0(x,y)e^{-\gamma z}$))

a) BVP : (9-53) + (9-61) ~ (9-64)

b) (9-54) ~ (9-60)

c) $(9\text{-}59)_{TM}$, $(9\text{-}60)_{TM}$ $\Rightarrow \begin{cases} E_z^0(x,y) = E_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \ (V/m) & (9\text{-}65) \\ h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2, \text{ for } m, n = 1, 2, 3 \cdots & (9\text{-}66) \end{cases}$

d) Faraday's law : $(\nabla \times \vec{E} = -j\omega\mu_0\vec{H})_x \Rightarrow \frac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu_0 H_x^0$

Ampere's law : $(\nabla \times \vec{H} = j\omega\epsilon_0\vec{E})_y \Rightarrow -\gamma H_x^0 - \underbrace{\cancel{\frac{\partial H_z^0}{\partial x}}}_{0} = j\omega\epsilon_0 E_y^0$

$\Longrightarrow$ eliminating E_y^0 with $h^2 = \gamma^2 + \omega^2\epsilon_0\mu_0 = \gamma^2 + k^2$

$$H_x^0 = \frac{j\omega\epsilon_0}{h^2}\frac{\partial E_z^0}{\partial y}$$

e) $\gamma^2 = h^2 - k^2 = h^2 - \omega^2\mu_0\epsilon_0 = h^2 - \frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \frac{\omega^2}{c^2}$

For propagating modes TM_{mn} ($f > f_c$),

$$\gamma = j\beta = j\sqrt{\frac{\omega^2}{c^2} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$\therefore \beta^2 = \frac{\omega^2}{c^2} - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{a/\sqrt{3}}\right)^2 = \frac{\omega^2}{c^2} - \frac{\pi^2}{a^2}(m^2 + 3n^2)$: dispersion eqn.

Cutoff at $\gamma = 0$

$\Rightarrow \frac{\omega_c^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \Rightarrow \left(\frac{2\pi f_c}{c}\right)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\Rightarrow (f_c)_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \underset{b = a/\sqrt{3}}{=} \frac{c}{2a}\sqrt{m^2 + 3n^2}$: cutoff frequency

f) Dominant mode $\Rightarrow$ smallest $(f_c)_{mn}$ $\Rightarrow$ TM_{11} mode

$(f_c)_{11} = \frac{c}{2a}\sqrt{1^2 + 3 \times 1^2} = \frac{c}{a} = \frac{3 \times 10^8}{a}$ (Hz)

g) $(\lambda_c)_{11} = \frac{c}{(f_c)_{11}} = \frac{c}{c/a} = a$ (m)

h) For $f > 1.2 (f_c)_{11}$,

$3.6 \times 10^9 > 1.2\,\frac{3 \times 10^8}{a} \Rightarrow a > 0.1$ (m)